Lecture 6

2025/2026

Microwave Devices and Circuits for Radiocommunications

2025/2026

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course or laboratory)**
- Lectures- **associate professor Radu Damian**
  - Tuesday **12-14**, **P2**
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 24.02.2026 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - **all materials/equipments authorized**

2025/2026

- Laboratory – **associate professor Radu Damian**
  - Monday 14-16, II.13 / (even weeks)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 24.02.2026)
    - personal homework

Lecture 6

- (Only) Lecture 6 - 24.03.2026
  - ~~12-14 P2~~
  - **16-18 ~~P6~~ online Teams**

General theory

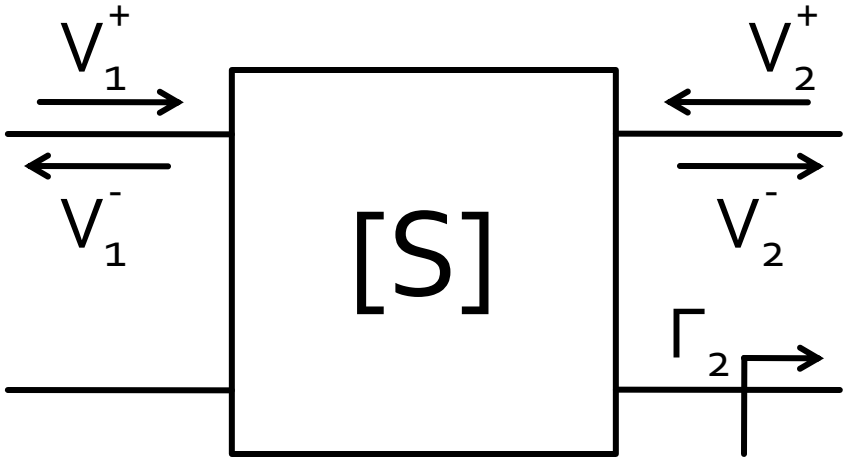
Microwave Network Analysis

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

Scattering matrix – S

- Scattering parameters



$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$$

- $V_2^+ = 0$ meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

Power waves for N ports

$$[b] = [F] \cdot \left([Z] - [Z_R]^* \right) \cdot \left([Z] + [Z_R] \right)^{-1} \cdot [F]^{-1} \cdot [a]$$

- The scattering matrix for power waves, $[S_p]$

$$[b] = [S_p] \cdot [a]$$

$$[S_p] = [F] \cdot \left([Z] - [Z_R]^* \right) \cdot \left([Z] + [Z_R] \right)^{-1} \cdot [F]^{-1}$$

- But: $[S] = \left([Z] - [Z_0] \right) \cdot \left([Z] + [Z_0] \right)^{-1}$

- Typically

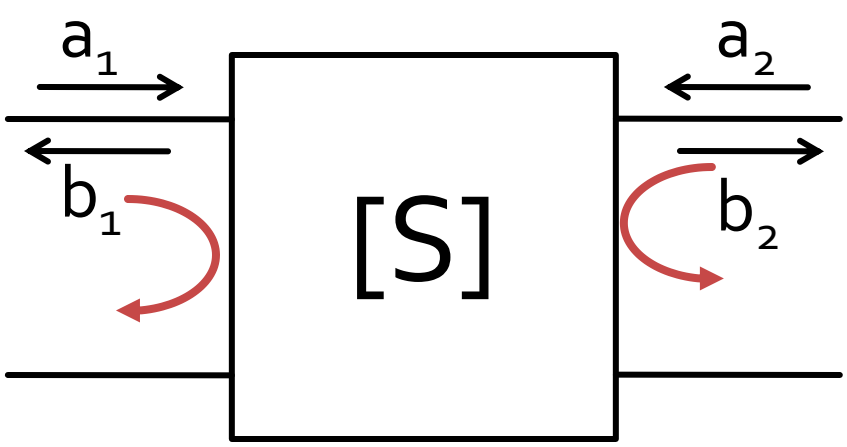
$$Z_{0i} = Z_{Ri} = R_0, \forall i$$

$$R_0 = 50\Omega$$

$$\boxed{[S_p] \equiv [S]}$$

- **they coincide!!!**

Scattering matrix – S

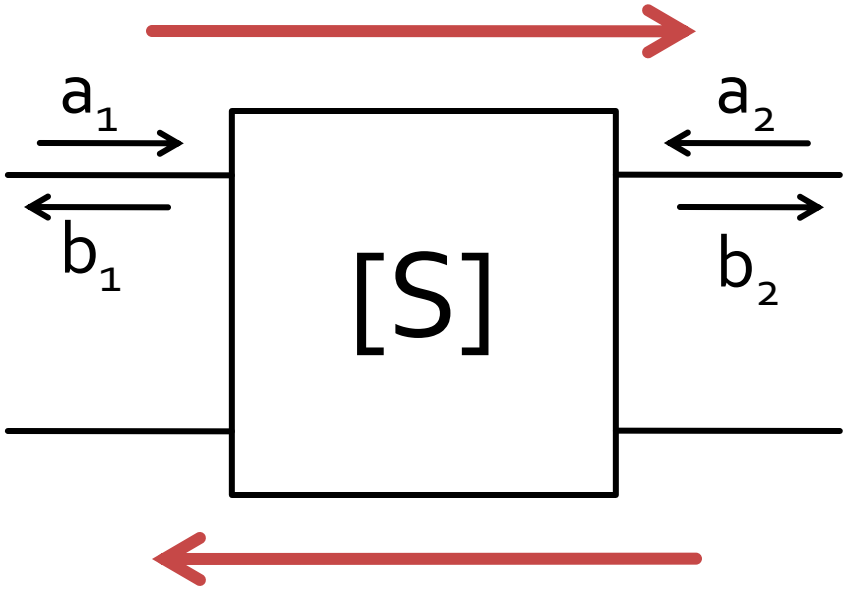


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$

- S_{11} and S_{22} are **reflection coefficients** at ports 1 and 2 when the other port is **matched**

Scattering matrix – S

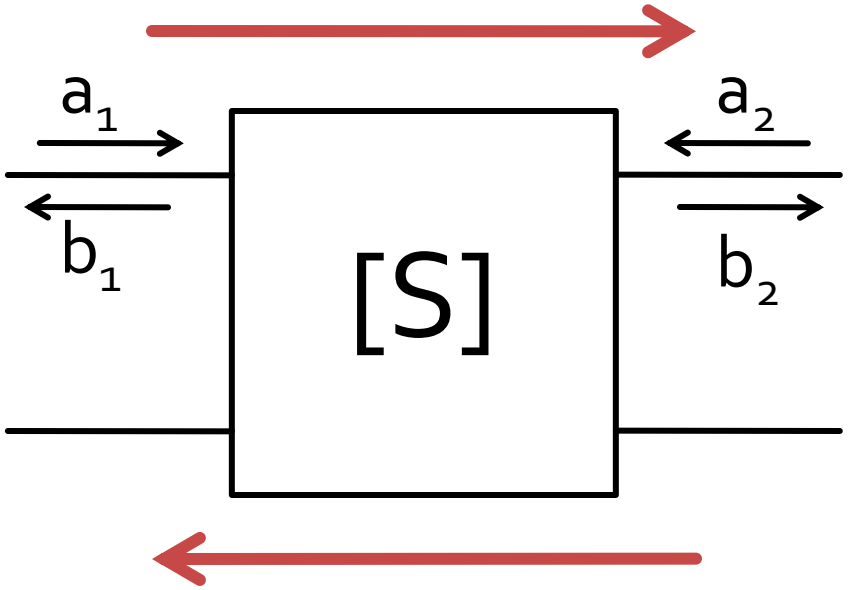


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \qquad S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$

- S_{21} si S_{12} are signal amplitude **gain** when the other port is **matched**

Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

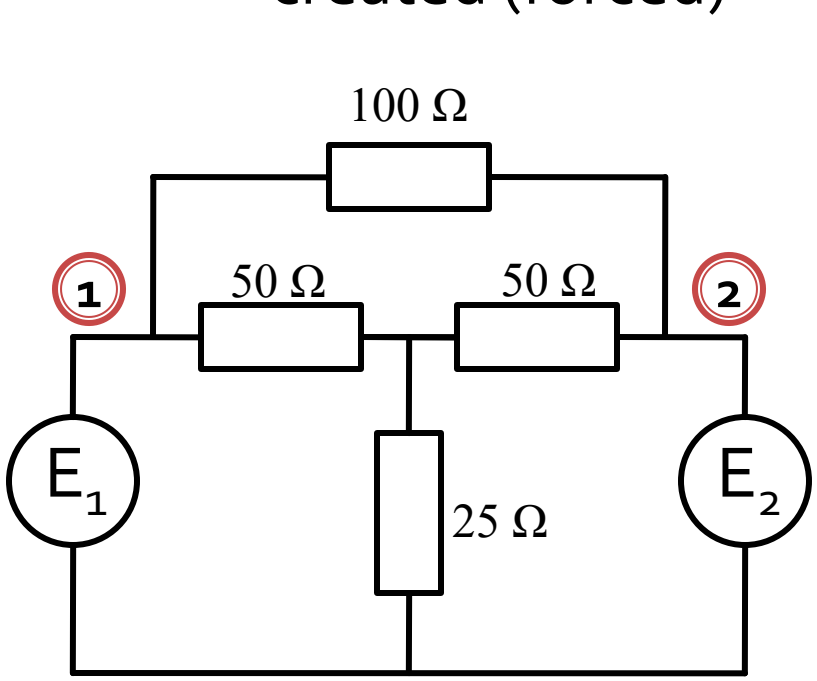
$$|S_{21}|^2 = \frac{Power\ in\ Z_0\ load}{Power\ \ from\ Z_0\ source}$$

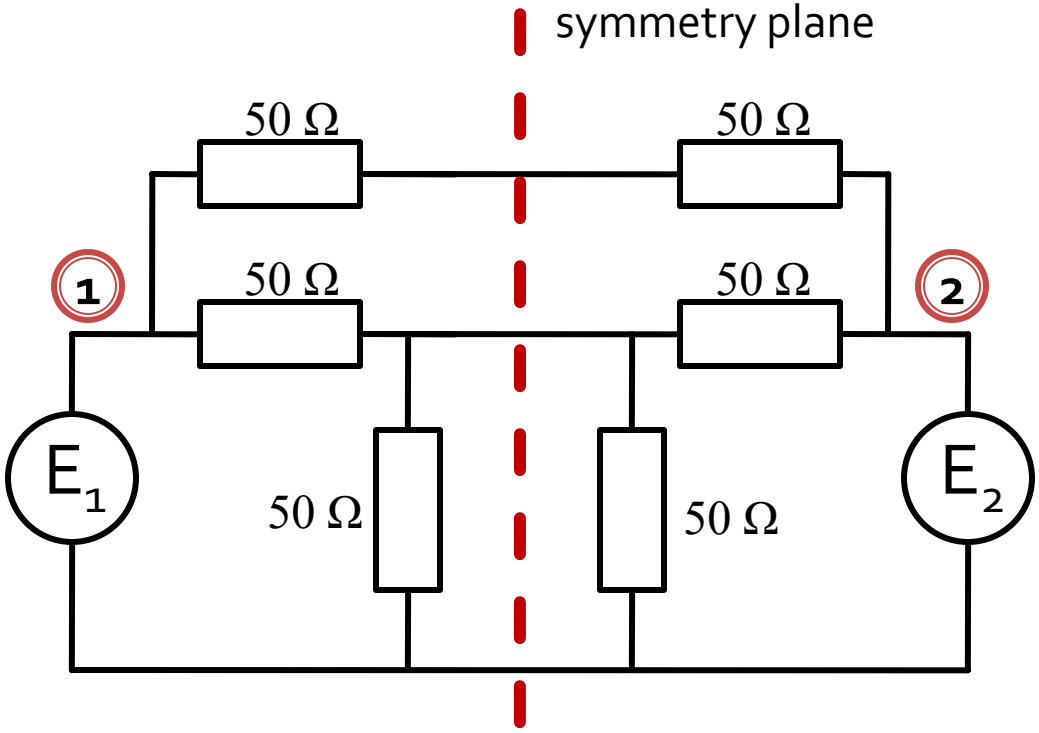
- a,b
  - information about signal power **AND** signal phase
- S_{ij}
  - network effect (gain) over signal power **including** phase information

Even/Odd Mode Analysis

Even/Odd Mode Analysis

- Even/Odd mode analysis benefit from the existence of symmetry planes in the circuit
  - existing or
  - created (forced)





Even/Odd Mode Analysis

- In linear circuits we can use the superposition principle
- advantages
  - reduction of the circuit complexity
  - decrease of the **number of ports** (**main** advantage)

Response (ODD + EVEN) = Response (ODD) + Response (EVEN)

We can benefit from existing symmetries !!

Power dividers and directional couplers

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~

Introduction

Power dividers and couplers

- Desired functionality:
  - division
  - combining
- of signal power

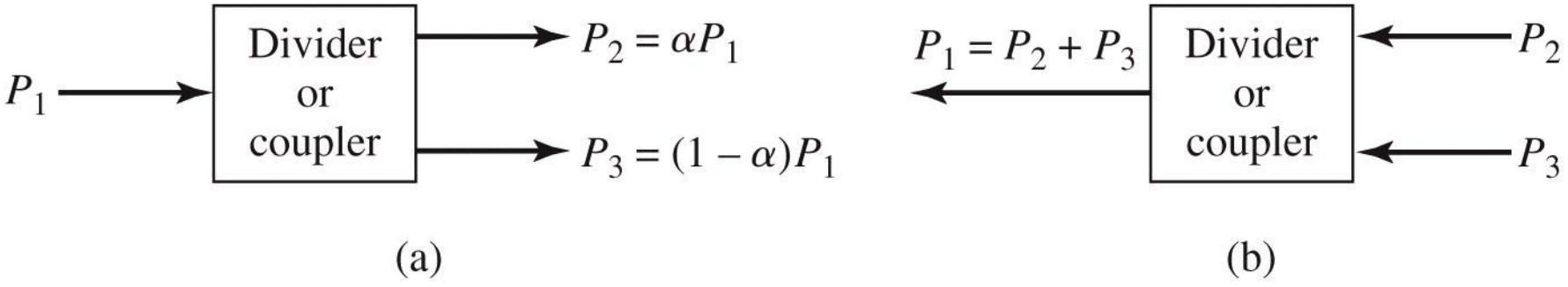
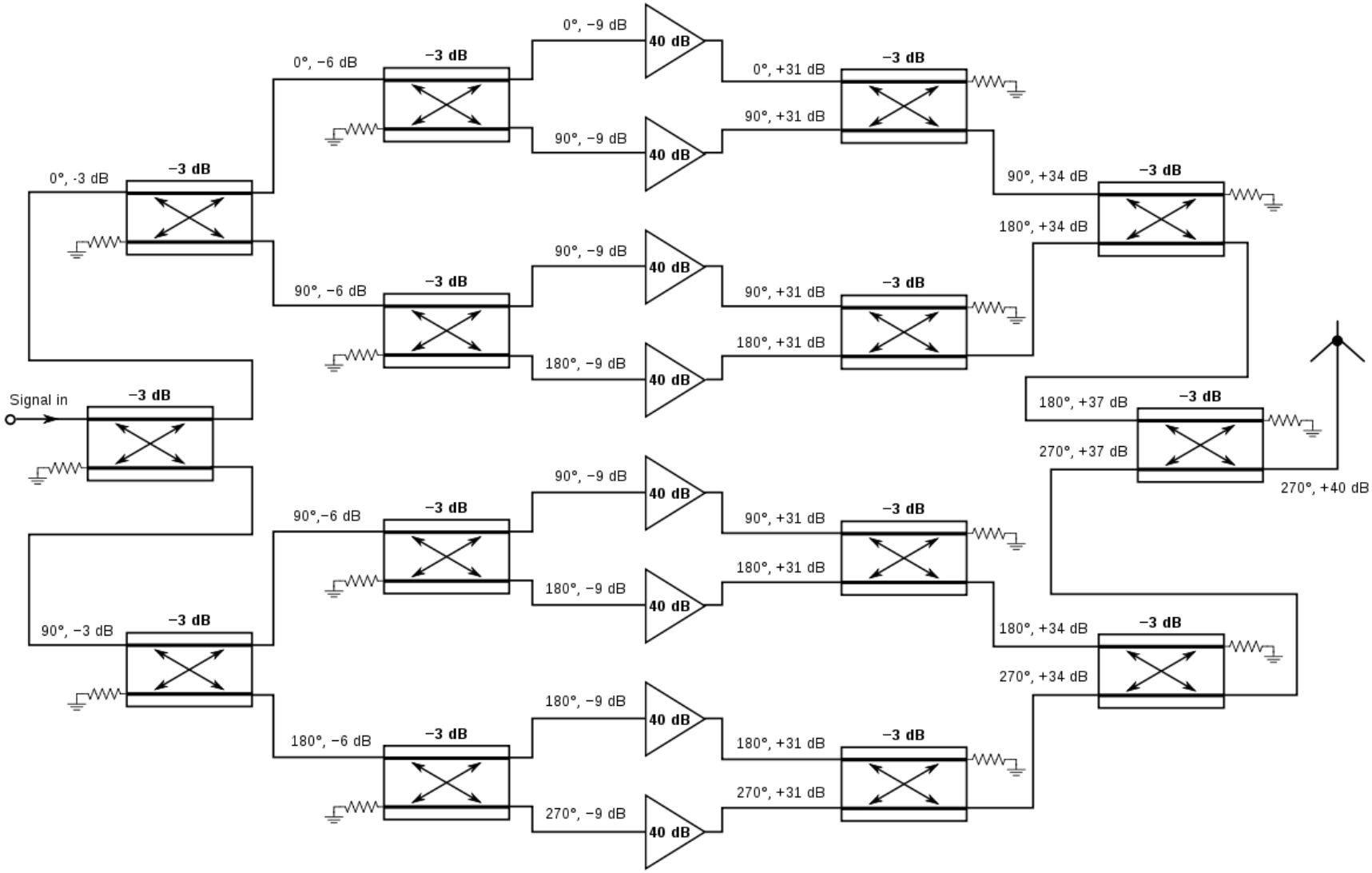


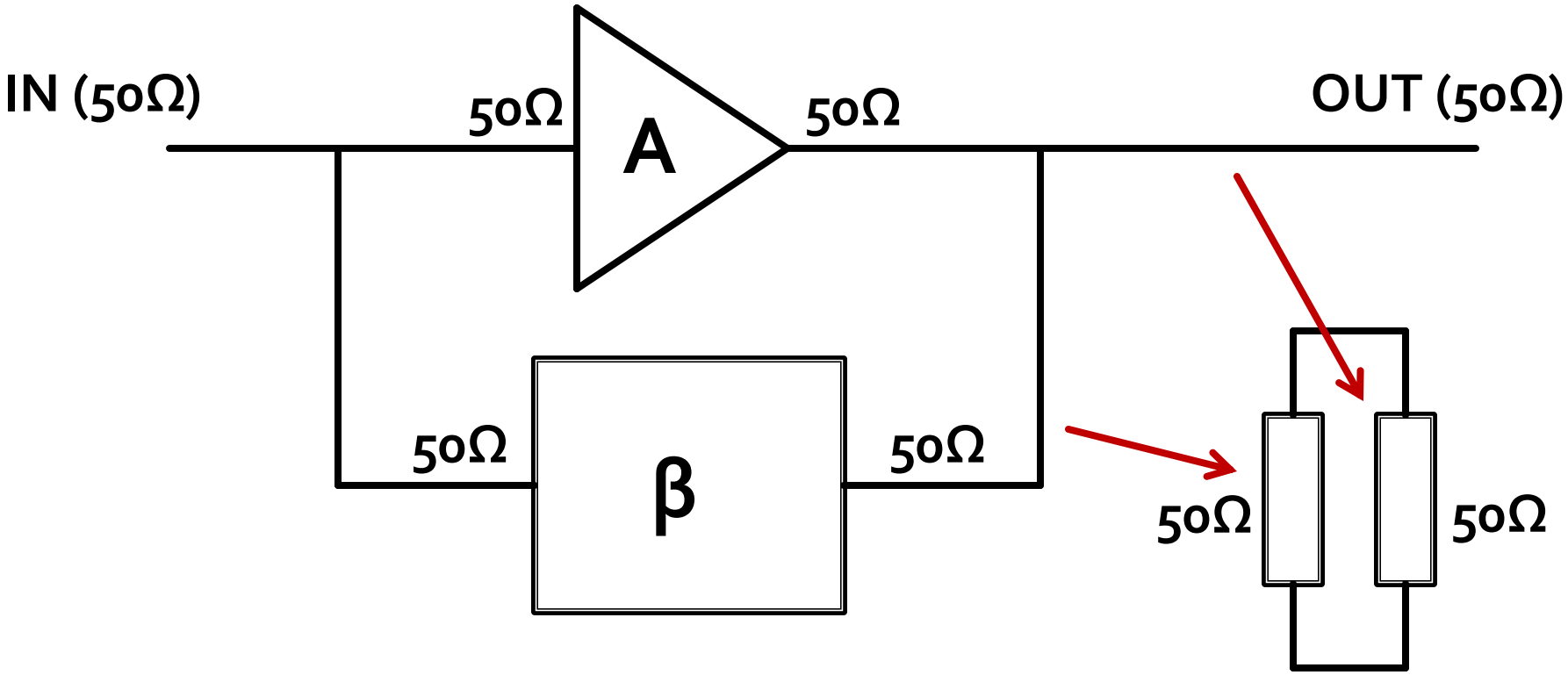
Figure 7.1
© John Wiley & Sons, Inc. All rights reserved.

Balanced amplifiers



Matching

- feedback amplifier



Three-Port Networks

- also known as T-Junctions
- characterized by a 3x3 **S** matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

- the device is **reciprocal** if it does **not** contain:
  - anisotropic materials (usually ferrites)
  - active circuits
- to avoid power loss, we would like to have a network that is:
  - **lossless**, and
  - **matched at all ports**
    - to avoid reflection power "loss"

Three-Port Networks

- reciprocal

$$[S] = [S]^t \qquad S_{ij} = S_{ji}, \forall j \neq i$$

$$S_{12} = S_{21}, S_{13} = S_{31}, S_{23} = S_{32}$$

- matched at all ports

$$S_{ii} = 0, \forall i \qquad S_{11} = 0, S_{22} = 0, S_{33} = 0$$

- then the S matrix is:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

Three-Port Networks

- reciprocal, matched at all ports, S matrix:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- lossless network
  - all the power injected in one port will be found exiting the network on all ports

$$[S]^* \cdot [S]^t = [1] \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = \delta_{ij}, \forall i, j$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{ki}^* = 1 \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = 0, \forall i \neq j$$

Three-Port Networks

- lossless network

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{ki}^{*} = 1$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{kj}^{*} = 0, \forall i \neq j$$

- 6 equations / 3 unknowns

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad S_{13}^{*} S_{23} = 0$$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \qquad S_{12}^{*} S_{13} = 0$$

$$|S_{13}|^2 + |S_{23}|^2 = 1 \qquad S_{23}^{*} S_{12} = 0$$

- **no solution** is possible

Three-Port Networks

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- 6 equations / 3 unknowns
  - no solution is possible
- A three-port network **cannot** be simultaneously:
  - reciprocal
  - lossless
  - matched at all ports
- If any one of these three conditions is relaxed, then a physically realizable device is possible

Nonreciprocal Three-Port Networks

- usually containing anisotropic materials, ferrites
- **nonreciprocal**, but matched at all ports and lossless $S_{ij} \neq S_{ji}$
- S matrix

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

- 6 equations / 6 unknowns

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad S_{31}^* S_{32} = 0$$
$$|S_{21}|^2 + |S_{23}|^2 = 1 \qquad S_{21}^* S_{23} = 0$$
$$|S_{31}|^2 + |S_{32}|^2 = 1 \qquad S_{12}^* S_{13} = 0$$

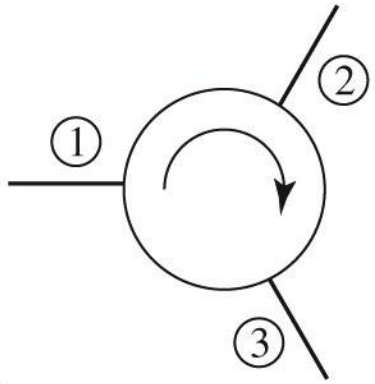
Nonreciprocal Three-Port Networks

- two possible solutions
- circulators
  - clockwise circulation

$$S_{12} = S_{23} = S_{31} = 0$$

$$|S_{21}| = |S_{32}| = |S_{13}| = 1$$

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



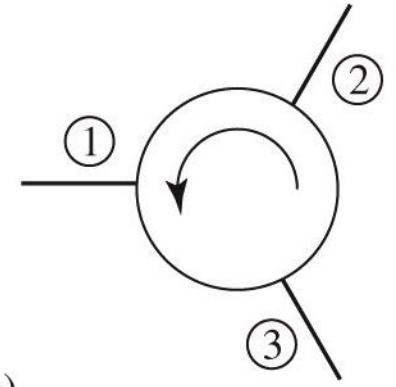
(a)

  - counterclockwise circulation

$$S_{21} = S_{32} = S_{13} = 0$$

$$|S_{12}| = |S_{23}| = |S_{31}| = 1$$

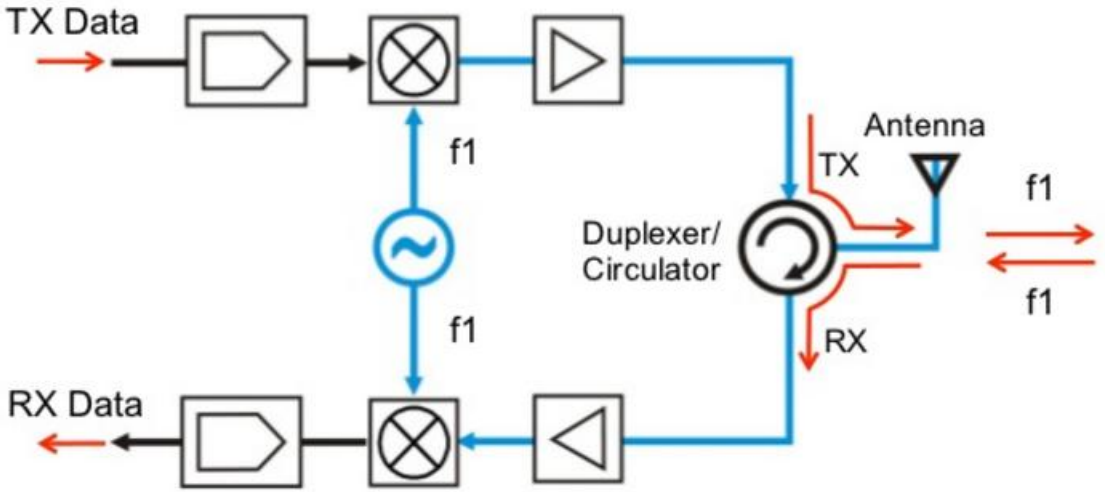
$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

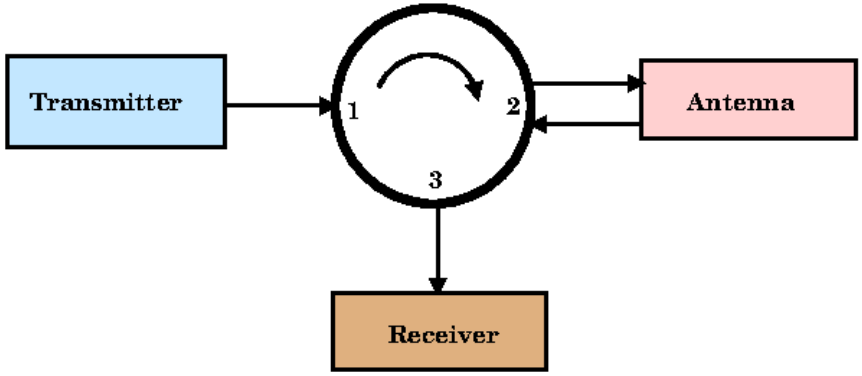


(b)

Nonreciprocal Three-Port Networks

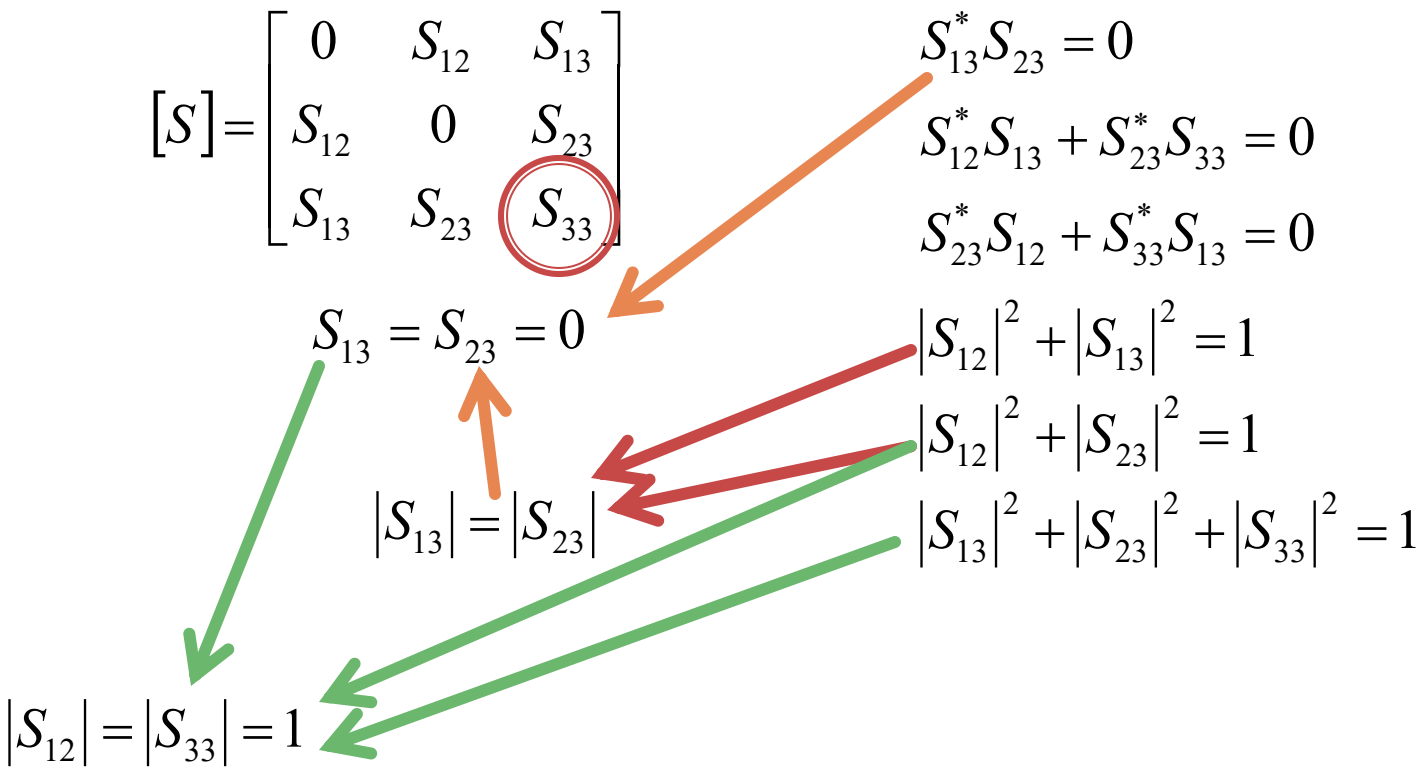
- circulator often found in duplexer





Mismatched Three-Port Networks

- A lossless and reciprocal three-port network can be matched only on two ports, eg. 1 and 2:



Mismatched Three-Port Networks

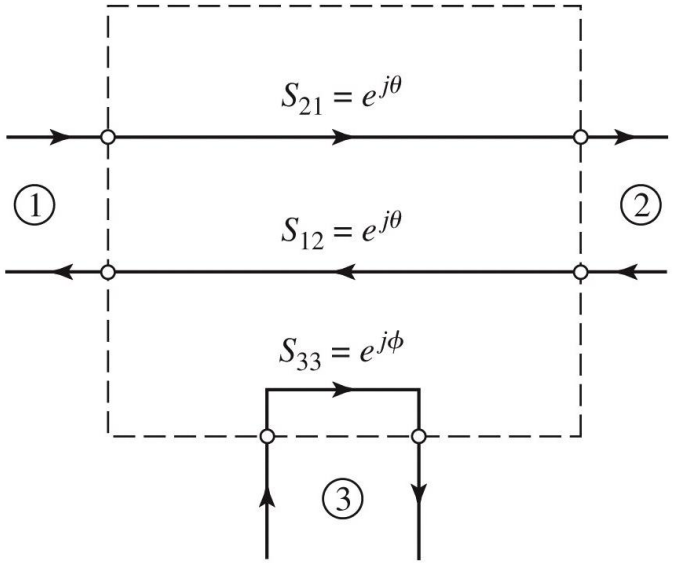
- A lossless and reciprocal three-port network

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

$$S_{13}=S_{23}=0 \qquad |S_{12}|=|S_{33}|=1$$

$$S_{12}=e^{j\theta} \qquad S_{33}=e^{j\varphi}$$

$$[S]=\begin{bmatrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\phi} \end{bmatrix}$$



- A lossless and reciprocal three-port network **degenerates** into two separate components:
  - a matched two-port **line**
  - a totally **mismatched one-port**:

Four-Port Networks

- characterized by a 4x4 **S** matrix

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

- the device is **reciprocal** if it does **not** contain:
  - anisotropic materials (usually ferrites)
  - active circuits
- to avoid power loss, we would like to have a network that is:
  - **lossless**, and
  - **matched at all ports**
    - to avoid reflection power "loss"

Four-Port Networks

- reciprocal

$$[S]=[S]^t \qquad S_{ij}=S_{ji},\forall j\neq i$$

$$S_{12}=S_{21},S_{13}=S_{31},S_{23}=S_{32}$$

- matched at all ports

$$S_{ii}=0,\forall i \qquad S_{11}=0,S_{22}=0,S_{33}=0,S_{44}=0$$

- then the S matrix is:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

Four-Port Networks

- reciprocal, matched at all ports, S matrix:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

- lossless network
  - all the power injected in one port will be found exiting the network on all ports

$$[S]^* \cdot [S]^t = [1] \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = \delta_{ij}, \forall i, j$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{ki}^* = 1 \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = 0, \forall i \neq j$$

Four-Port Networks

$$S_{13}^* \cdot S_{23} + S_{14}^* \cdot S_{24} = 0 \quad /\cdot S_{24}^*$$
$$S_{14}^* \cdot S_{13} + S_{24}^* \cdot S_{23} = 0 \quad /\cdot S_{13}^*$$
$$S_{14}^* \cdot \left(|S_{13}|^2 - |S_{24}|^2 \right) = 0$$

$$S_{12}^* \cdot S_{23} + S_{14}^* \cdot S_{34} = 0 \quad /\cdot S_{12}$$
$$S_{14}^* \cdot S_{12} + S_{34}^* \cdot S_{23} = 0 \quad /\cdot S_{34}^*$$
$$S_{23} \cdot \left(|S_{12}|^2 - |S_{34}|^2 \right) = 0$$

- one solution: $S_{14} = S_{23} = 0$
- resulting coupler is **directional**

$$|S_{12}|^2 + |S_{13}|^2 = 1$$
$$|S_{12}|^2 + |S_{24}|^2 = 1$$
$$|S_{13}|^2 + |S_{34}|^2 = 1$$
$$|S_{24}|^2 + |S_{34}|^2 = 1$$

$$|S_{13}| = |S_{24}|$$
$$|S_{12}| = |S_{34}|$$

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

Four-Port Networks

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$|S_{12}|=|S_{34}|=\alpha \qquad |S_{13}|=|S_{24}|=\beta$$

β – voltage coupling coefficient

- We can choose the phase reference

$$S_{12}=S_{34}=\alpha \qquad S_{13}=\beta\cdot e^{j\theta} \qquad S_{24}=\beta\cdot e^{j\phi}$$

$$S_{12}^{*}\cdot S_{13}+S_{24}^{*}\cdot S_{34}=0 \quad \rightarrow \quad \theta+\phi=\pi\pm 2\cdot n\cdot\pi$$

$$|S_{12}|^{2}+|S_{24}|^{2}=1 \quad \rightarrow \quad \alpha^{2}+\beta^{2}=1$$

- The other possible solution for previous equations offer either essentially the same result (with a different phase reference) or the degenerate case (2 separate two port networks side by side)

$$S_{14}^{*}\cdot\left(|S_{13}|^{2}-|S_{24}|^{2}\right)=0 \qquad S_{23}\cdot\left(|S_{12}|^{2}-|S_{34}|^{2}\right)=0$$

Four-Port Networks

- A four-port network simultaneously:
  - matched at all ports
  - reciprocal
  - lossless
- is **always directional**
  - the signal power injected into one port is transmitted **only towards two** of the other three ports

$$[S]=\begin{bmatrix} 0 & \alpha & \beta\cdot e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta\cdot e^{j\phi} \\ \beta\cdot e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta\cdot e^{j\phi} & \alpha & 0 \end{bmatrix}$$

Four-Port Networks

- two particular choices commonly occur in practice
  - A Symmetric Coupler (90°) $\theta = \phi = \pi/2$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

  - An Antisymmetric Coupler (180°) $\theta = 0, \phi = \pi$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Directional Coupler

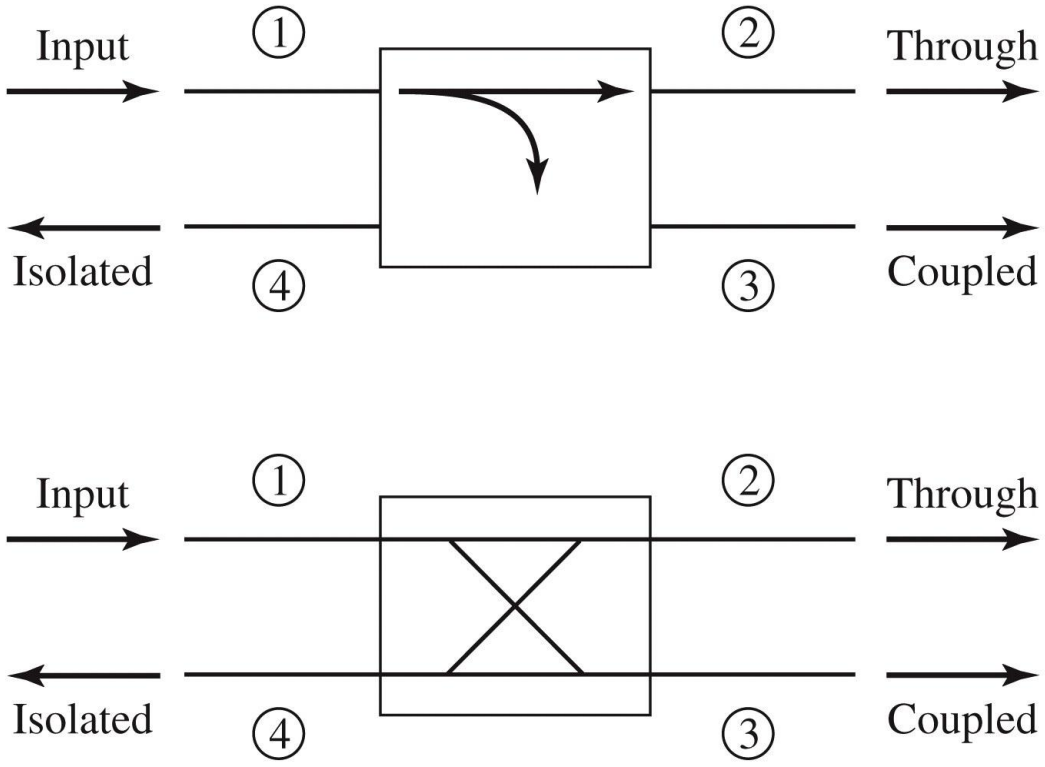


Figure 7.4
© John Wiley & Sons, Inc. All rights reserved.

$$|S_{12}|^2 = \alpha^2 = 1 - \beta^2$$

$$|S_{13}|^2 = \beta^2$$

Coupling

$$C = 10\log\frac{P_1}{P_3} = -20\cdot\log(\beta)\,[\text{dB}]$$

Directivity

$$D = 10\log\frac{P_3}{P_4} = 20\cdot\log\left(\frac{\beta}{|S_{14}|}\right)[\text{dB}]$$

Isolation

$$I = 10\log\frac{P_1}{P_4} = -20\cdot\log|S_{14}|\,[\text{dB}]$$

$$I = D + C\,,\ [dB]$$

Power dividers

Three-Port Networks

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- 6 equations / 3 unknowns
  - no solution is possible
- A three-port network **cannot** be simultaneously:
  - reciprocal
  - **lossless**
  - matched at all ports
- If any one of these three conditions is relaxed, then a physically realizable device is possible

Power division of the T-junction

- consists in splitting an input line into two separate output lines
- available in various technologies for the lines

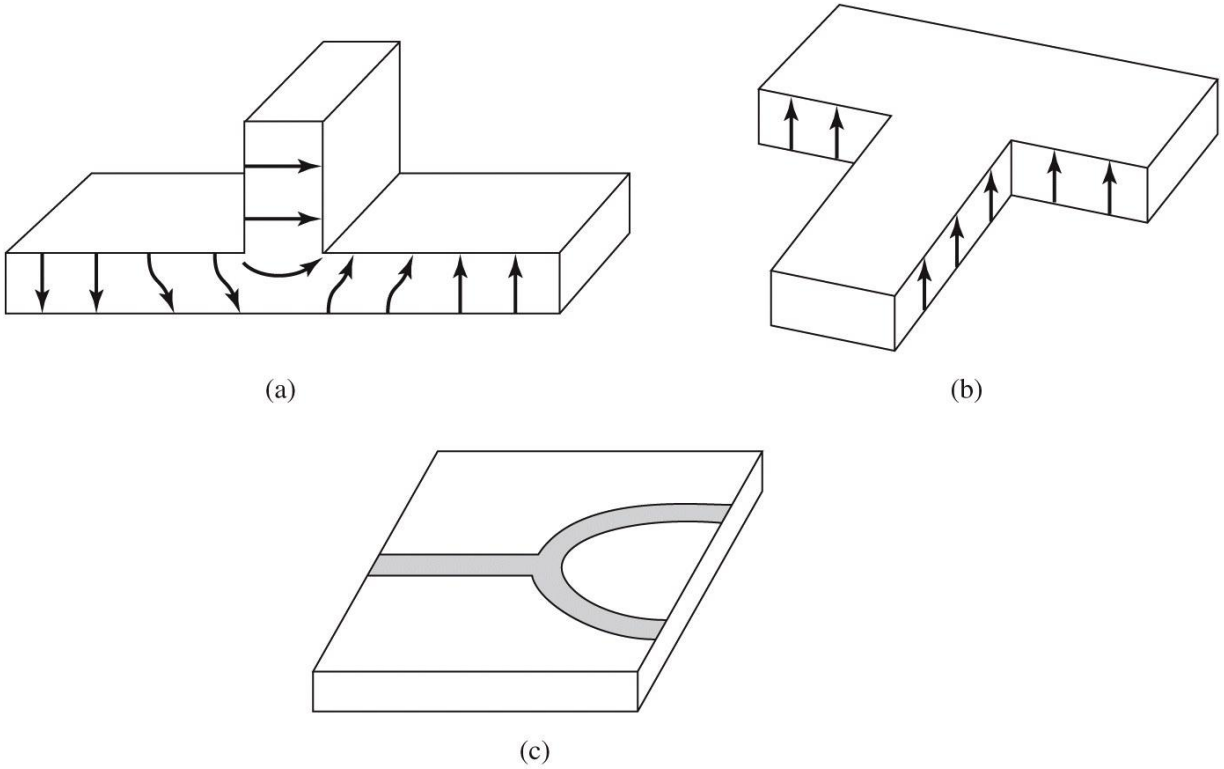


Figure 7.5

© John Wiley & Sons, Inc. All rights reserved.

Power division of the T-junction

- if the lines are lossless, the network is reciprocal, so it cannot be matched at all ports simultaneously

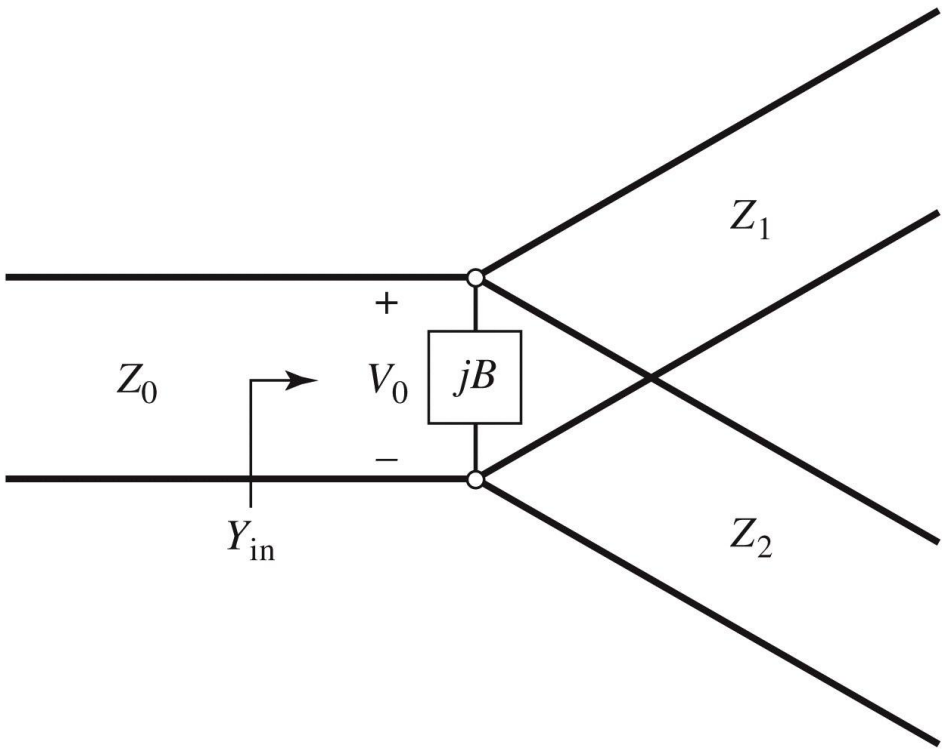


Figure 7.6
© John Wiley & Sons, Inc. All rights reserved.

- there may be fringing fields and higher order modes associated with the discontinuity at such a junction
- the stored energy can be accounted for by a lumped susceptance: B
- Designing the power divider targets matching to the input line Z_0
  - outputs (unmatched, Z_1 and Z_2) can be, if needed, matched to Z_0 ($\lambda/4$, binomial, Chebyshev)

Power division of the T-junction

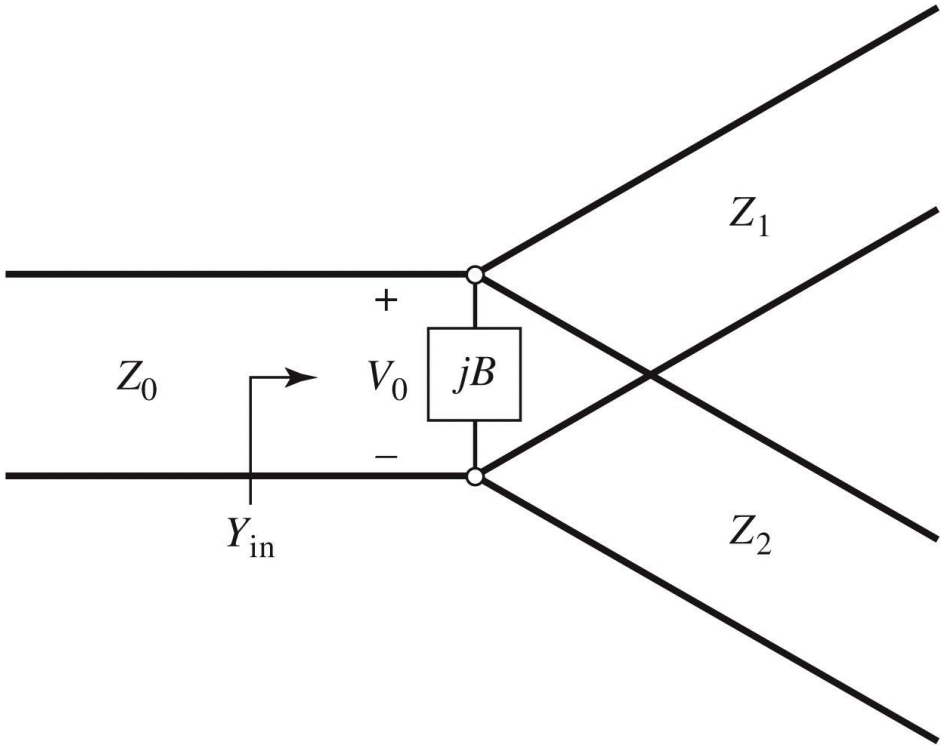


Figure 7.6
© John Wiley & Sons, Inc. All rights reserved.

$$Y_{in} = j \cdot B + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}$$

- If the transmission lines are assumed to be lossless, then the characteristic impedances are real
- the matching condition can be met only if **B ≅ o** thus the matching condition is:

$$\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}$$

In practice, if **B** is not negligible, some type of discontinuity compensation or a reactive tuning element can usually be used to cancel this susceptance, at least over a narrow frequency range.

Power division of the T-junction

- if V_0 is the voltage at the junction, we can compute how the input power is divided between the two output lines

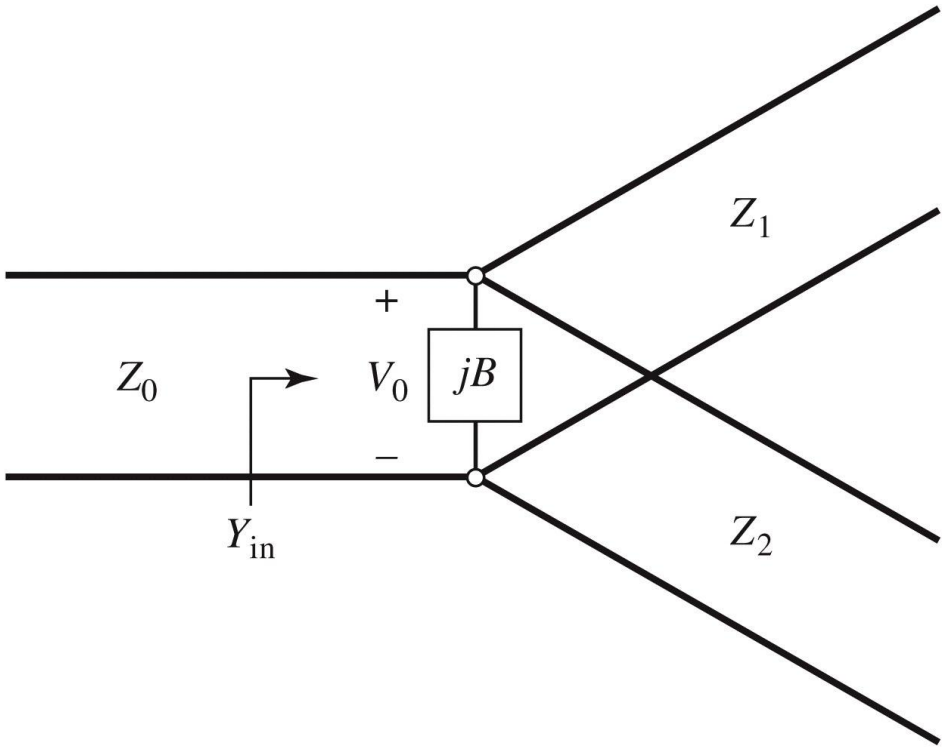


Figure 7.6
© John Wiley & Sons, Inc. All rights reserved.

$$P_{in} = \frac{1}{2} \cdot \frac{V_0^2}{Z_0}$$

$$P_1 = \frac{1}{2} \cdot \frac{V_0^2}{Z_1}$$

$$P_2 = \frac{1}{2} \cdot \frac{V_0^2}{Z_2}$$

then:

$$P_{in} = P_1 + P_2$$ (lossless/input matching)

$$\frac{P_1}{P_2} = \frac{Z_2}{Z_1} = \alpha$$ (power division between the two output lines)

$$P_1 = P_{in} \cdot \frac{Z_2}{Z_1 + Z_2} \qquad P_2 = P_{in} \cdot \frac{Z_1}{Z_1 + Z_2}$$

$$P_1 = P_{in} \cdot \frac{\alpha}{1+\alpha} \qquad P_2 = P_{in} \cdot \frac{1}{1+\alpha}$$

$$Z_1 = Z_0 \cdot \left(1 + \frac{1}{\alpha}\right) \qquad Z_2 = Z_0 \cdot (1+\alpha)$$

Power division of the T-junction

- S matrix
  - lossless (unitary matrix)
  - reciprocal (symmetrical matrix)
  - input port is matched $S_{11} = 0$

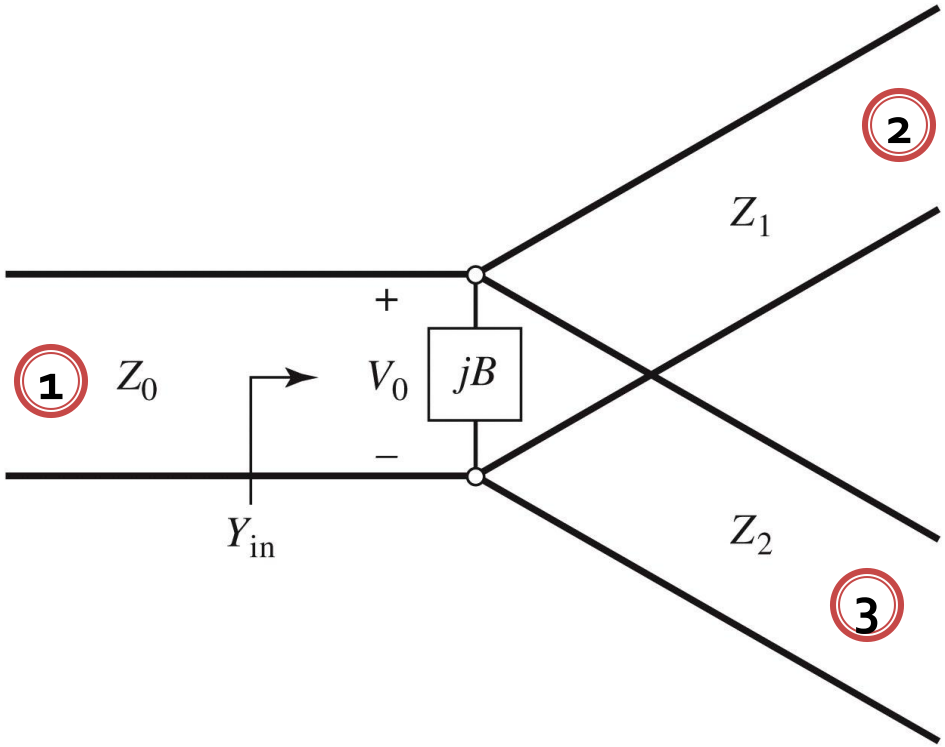


Figure 7.6
© John Wiley & Sons, Inc. All rights reserved.

$$P_2 = P_1 \cdot \frac{\alpha}{1+\alpha} \qquad S_{21} = S_{12} = \sqrt{\frac{\alpha}{1+\alpha}}$$

$$P_3 = P_1 \cdot \frac{1}{1+\alpha} \qquad S_{31} = S_{13} = \sqrt{\frac{1}{1+\alpha}}$$

the reflection coefficients seen looking into the output ports

$$S_{22} = \Gamma_1 = \frac{Z_0 \,\|\, Z_2 - Z_1}{Z_0 \,\|\, Z_2 + Z_1} = -\frac{1}{1+\alpha}$$

$$S_{33} = \Gamma_2 = \frac{Z_0 \,\|\, Z_1 - Z_2}{Z_0 \,\|\, Z_1 + Z_2} = -\frac{\alpha}{1+\alpha}$$

Power division of the T-junction

$$[S]=\begin{bmatrix} 0 & \sqrt{\frac{\alpha}{1+\alpha}} & \sqrt{\frac{1}{1+\alpha}} \\ \sqrt{\frac{\alpha}{1+\alpha}} & -\frac{1}{1+\alpha} & x \\ \sqrt{\frac{1}{1+\alpha}} & x & -\frac{\alpha}{1+\alpha} \end{bmatrix}$$

Unitary matrix, columns 1 and 2

$$0-\frac{1}{1+\alpha}\cdot\sqrt{\frac{\alpha}{1+\alpha}}+x\cdot\sqrt{\frac{1}{1+\alpha}}=0$$

$$S_{23}=S_{32}=\frac{\sqrt{\alpha}}{1+\alpha}$$

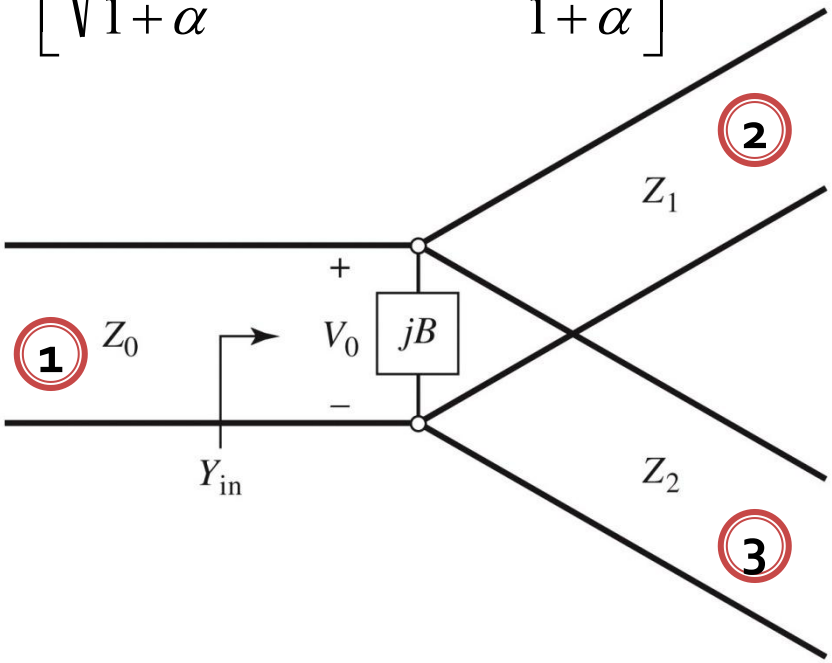


Figure 7.6
© John Wiley & Sons, Inc. All rights reserved.

$$[S]=\begin{bmatrix} 0 & \sqrt{\frac{\alpha}{1+\alpha}} & \sqrt{\frac{1}{1+\alpha}} \\ \sqrt{\frac{\alpha}{1+\alpha}} & -\frac{1}{1+\alpha} & \frac{\sqrt{\alpha}}{1+\alpha} \\ \sqrt{\frac{1}{1+\alpha}} & \frac{\sqrt{\alpha}}{1+\alpha} & -\frac{\alpha}{1+\alpha} \end{bmatrix}$$

Power division of the T-junction

- 3dB divider
  - equal splitting of the power between the two outputs
  - $Z_1 = Z_2 = 2 \cdot Z_0$, $\alpha = 1$

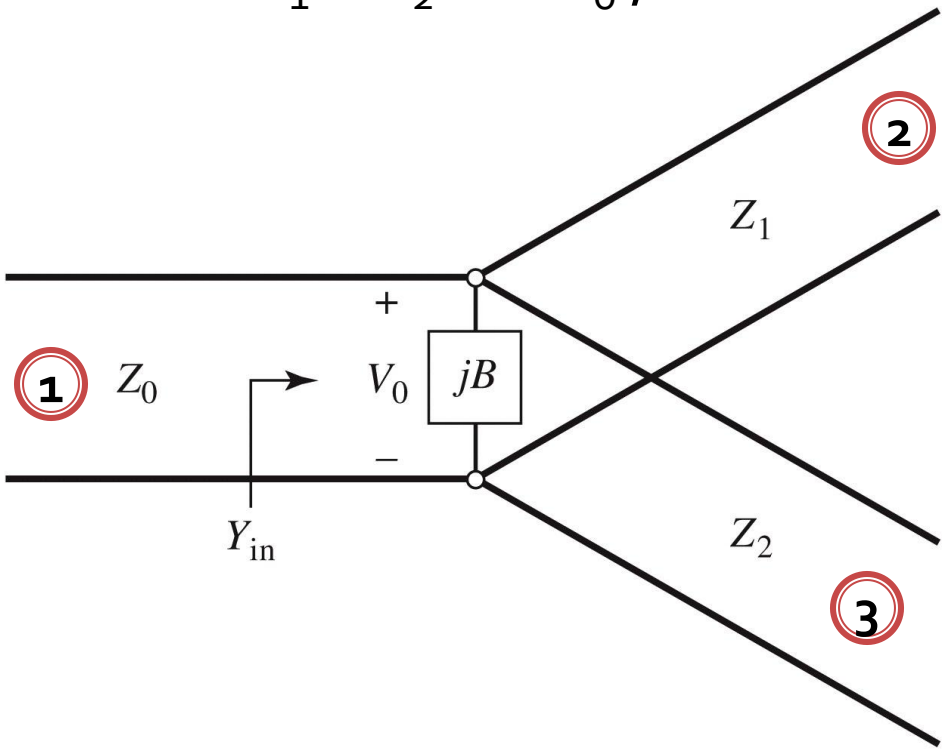


Figure 7.6
© John Wiley & Sons, Inc. All rights reserved.

$$[S] = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

If we add $\lambda/4$ transformers to match outputs to Z_0 S matrix:

$$[S] = \begin{bmatrix} 0 & -\frac{j}{\sqrt{2}} & -\frac{j}{\sqrt{2}} \\ -\frac{j}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{j}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Example

- Design a lossless T-junction divider with a 30Ω source impedance to give a 3:1 power split. Design quarter-wave matching transformers to convert the impedances of the output lines to 30Ω. (**Pozar** problem)

$$P_{in} = \frac{1}{2} \cdot \frac{V_0^2}{Z_0} \qquad \begin{cases} P_1 + P_2 = P_{in} \\ P_1 : P_2 = 3:1 \end{cases} \Rightarrow \begin{cases} P_1 = \frac{1}{4} \cdot P_{in} \\ P_2 = \frac{3}{4} \cdot P_{in} \end{cases}$$

$$P_1 = \frac{1}{2} \cdot \frac{V_0^2}{Z_1} = \frac{1}{4} \cdot P_{in} \qquad Z_1 = 4 \cdot Z_0 = 120\,\Omega$$

$$P_2 = \frac{1}{2} \cdot \frac{V_0^2}{Z_2} = \frac{3}{4} \cdot P_{in} \qquad Z_2 = 4 \cdot Z_0 / 3 = 40\,\Omega$$

Input match check

$$Z_{in} = 40\,\Omega \,||\, 120\,\Omega = 30\,\Omega$$

quarter-wave transformers $Z_c^i = \sqrt{Z_i \cdot Z_L}$

$$Z_c^1 = \sqrt{Z_1 \cdot Z_L} = \sqrt{120\Omega \cdot 30\Omega} = 60\Omega \qquad Z_c^2 = \sqrt{Z_2 \cdot Z_L} = \sqrt{40\Omega \cdot 30\Omega} = 34.64\Omega$$

Resistive Divider

- If a three-port divider contains lossy components, it can be made to be :
  - reciprocal
  - matched at all ports

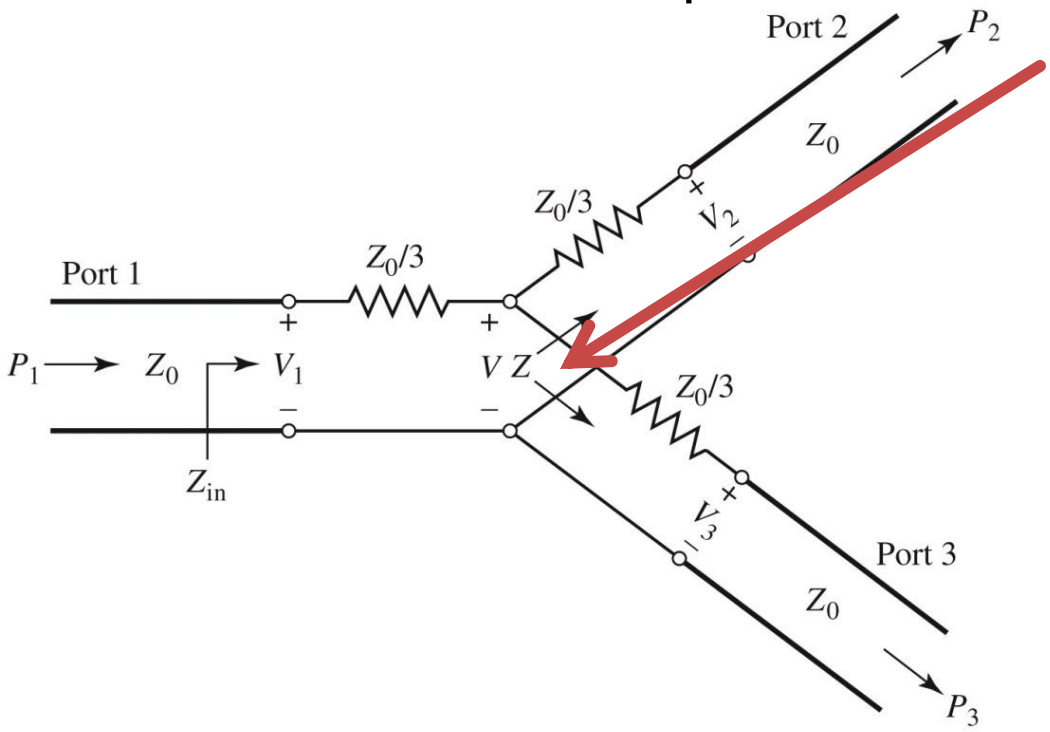


Figure 7.7
© John Wiley & Sons, Inc. All rights reserved.

The impedance Z, seen looking into the Zo/3 resistor followed by a terminated output line:

$$Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3}$$

The input line will be terminated with a Zo/3 resistor in series with two such lines Z in parallel

$$Z_{in} = \frac{Z_0}{3} + \frac{1}{2} \cdot \frac{4Z_0}{3} = Z_0$$

so it will be matched: $S_{11} = 0$

from symmetry: $S_{11} = S_{22} = S_{33} = 0$

Resistive Divider

- If a three-port divider contains lossy components, it can be made to be :
  - reciprocal
  - matched at all ports $S_{11} = S_{22} = S_{33} = 0$

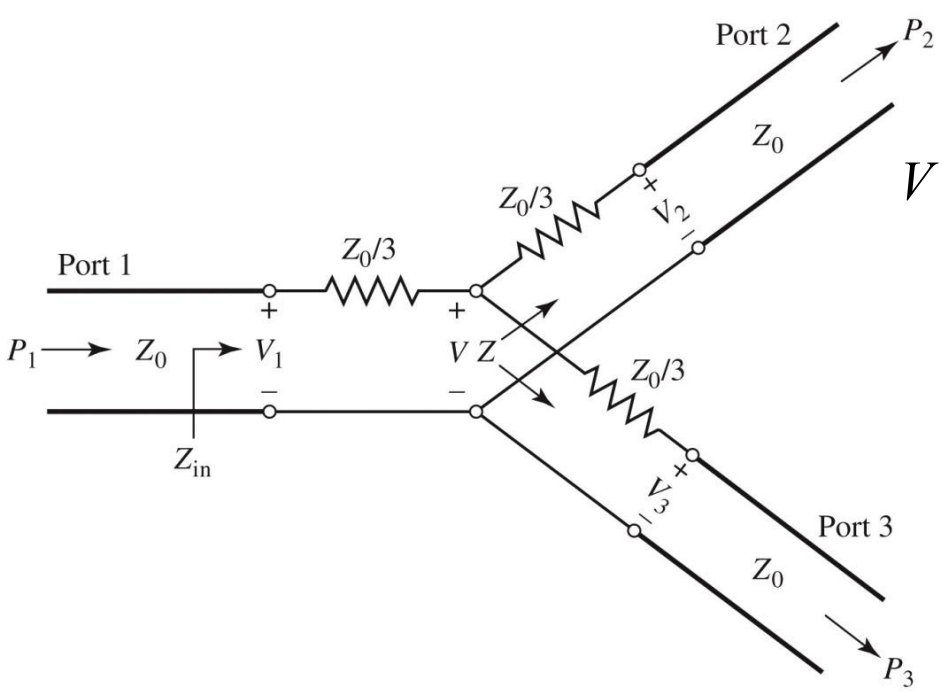


Figure 7.7
© John Wiley & Sons, Inc. All rights reserved.

If the voltage at port 1 is V1, then by voltage division the voltage V at the junction is:

$$V = V_1 \cdot \frac{Z/2}{Z/2 + Z_0/3} = V_1 \cdot \frac{2Z_0/3}{2Z_0/3 + Z_0/3} = \frac{2}{3} \cdot V_1$$

The output voltages are, again by voltage division :

$$V_2 = V_3 = V \cdot \frac{Z_0}{Z_0 + Z_0/3} = \frac{3}{4} \cdot V = \frac{1}{2} \cdot V_1$$

$$S_{21} = S_{31} = \frac{1}{2}$$

from symmetry: $S_{21} = S_{31} = S_{23} = \frac{1}{2}$

Resistive Divider

- If a three-port divider contains lossy components, it can be made to be :
  - reciprocal (S matrix is symmetrical) $S_{21} = S_{31} = S_{23} = \frac{1}{2}$
  - matched at all ports $S_{11} = S_{22} = S_{33} = 0$

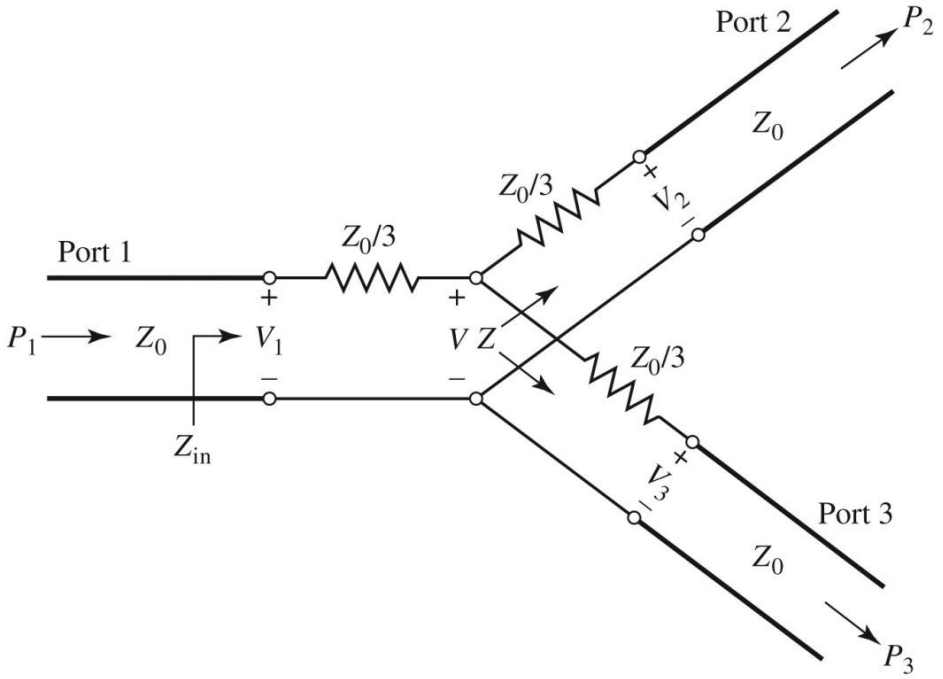


Figure 7.7
© John Wiley & Sons, Inc. All rights reserved.

S matrix: $$[S] = \frac{1}{2} \cdot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

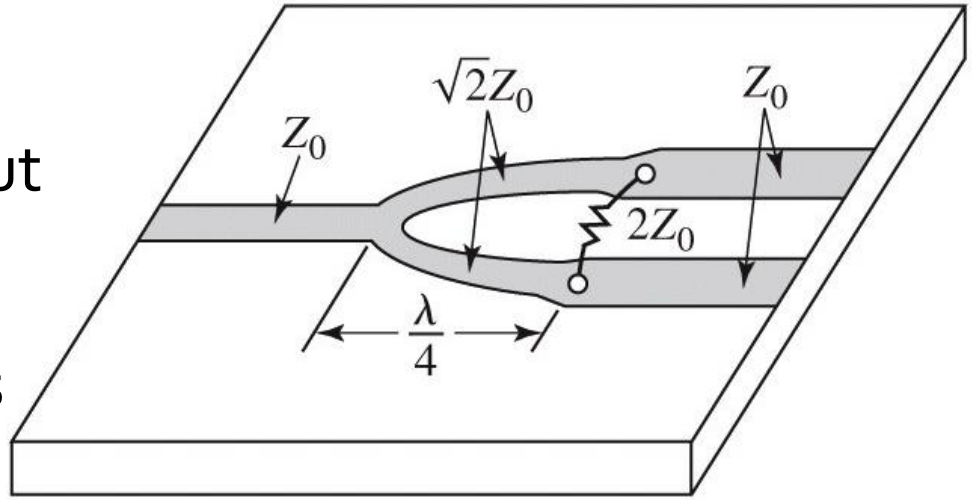
Powers: $$P_{in} = \frac{1}{2} \cdot \frac{V_1^2}{Z_0}$$

$$P_2 = P_3 = \frac{1}{2} \cdot \frac{(1/2 V_1)^2}{Z_0} = \frac{1}{8} \cdot \frac{V_1^2}{Z_0} = \frac{1}{4} \cdot P_{in}$$

Half of the supplied power is dissipated in the 3 resistors. The output powers are 6 dB below the input power level

The Wilkinson power divider

- Previous power dividers suffer from a major drawback, there is not isolation between the two output ports $S_{23} = S_{32} \neq 0$
  - this requirement is important in some applications
- The Wilkinson power divider solves this problem
  - it also has the useful property of appearing **lossless** when the output ports are matched
  - only reflected power from the output ports is dissipated



The Wilkinson power divider

- one input line
- two $\lambda/4$ transformers
- one resistor between the output lines

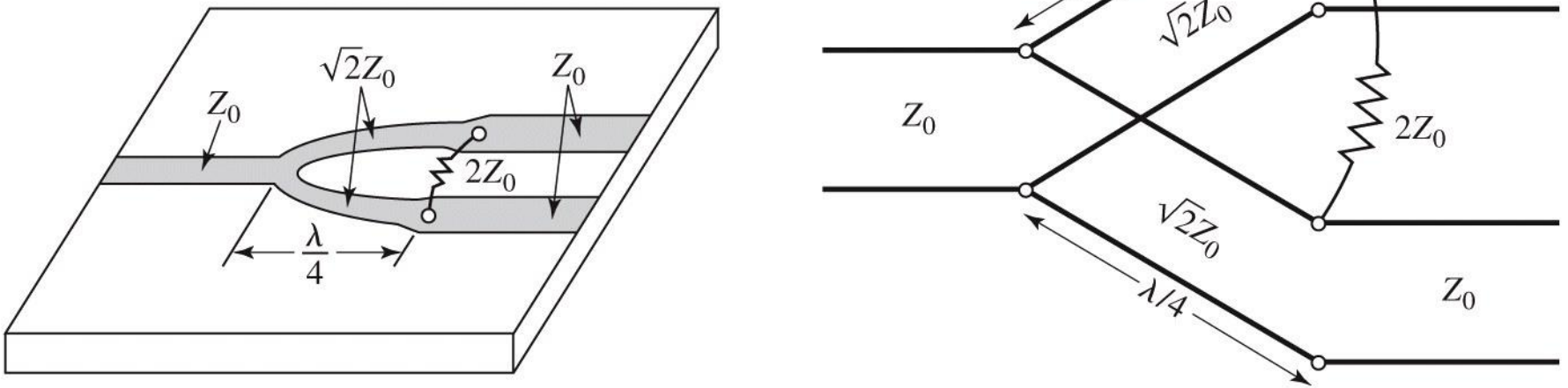


Figure 7.8

© John Wiley & Sons, Inc. All rights reserved.

Even/Odd Mode Analysis

- In linear circuits we can use the superposition principle
- advantages
  - reduction of the circuit complexity
  - decrease of the number of ports (**main** advantage)

Response (ODD + EVEN) = Response (ODD) + Response (EVEN)

We can benefit from existing symmetries !!

The Wilkinson power divider

- the circuit in normalized and symmetric form

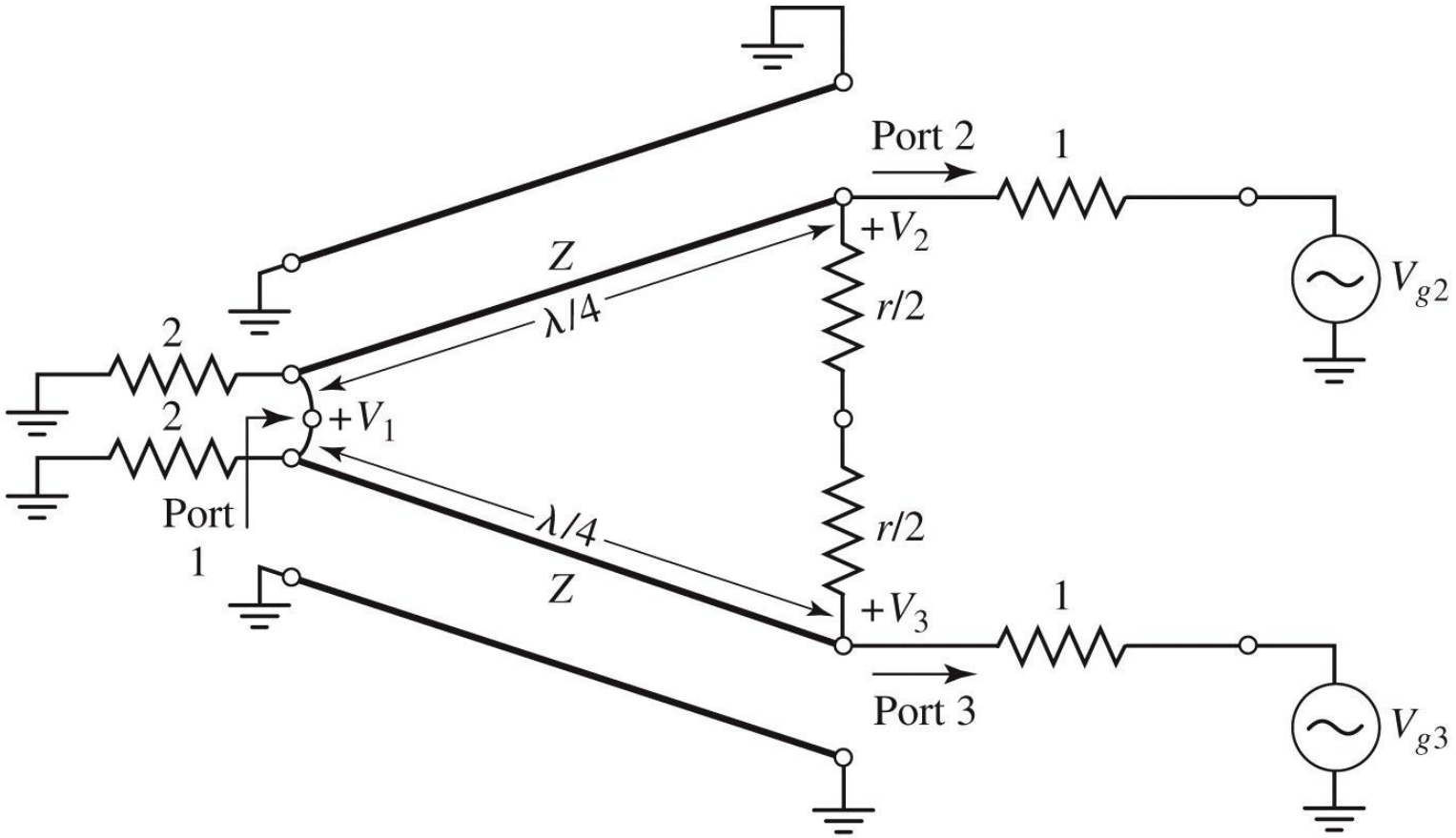
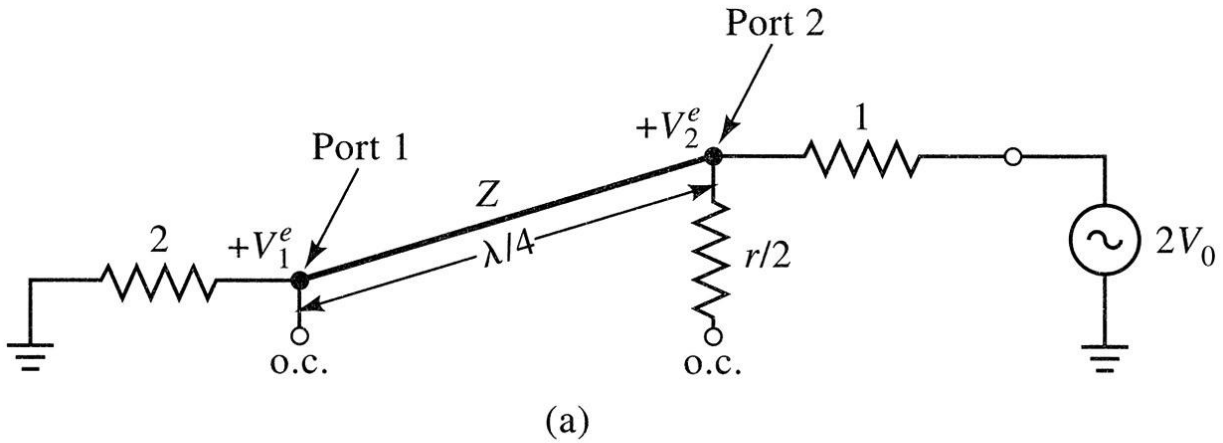


Figure 7.9

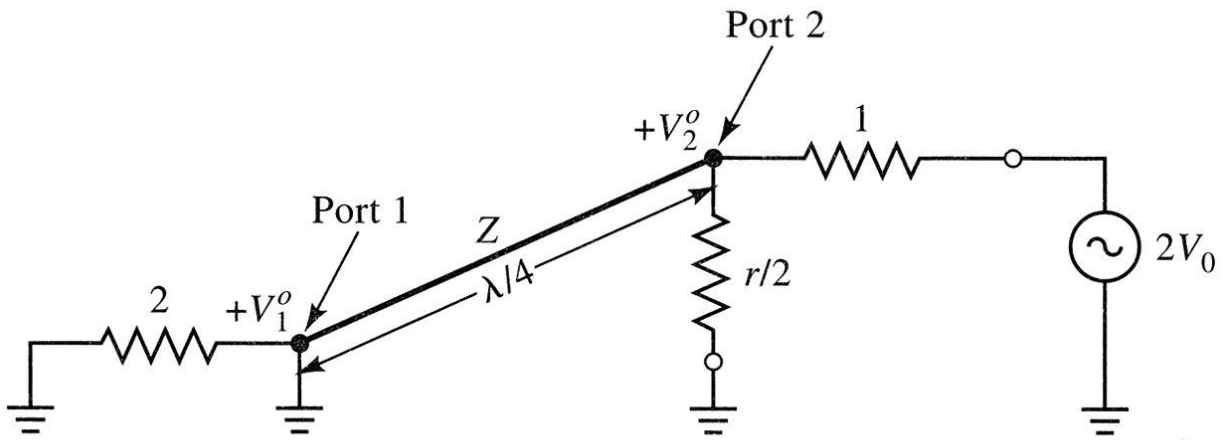
© John Wiley & Sons, Inc. All rights reserved.

The Wilkinson power divider

- Even/Odd Mode Analysis



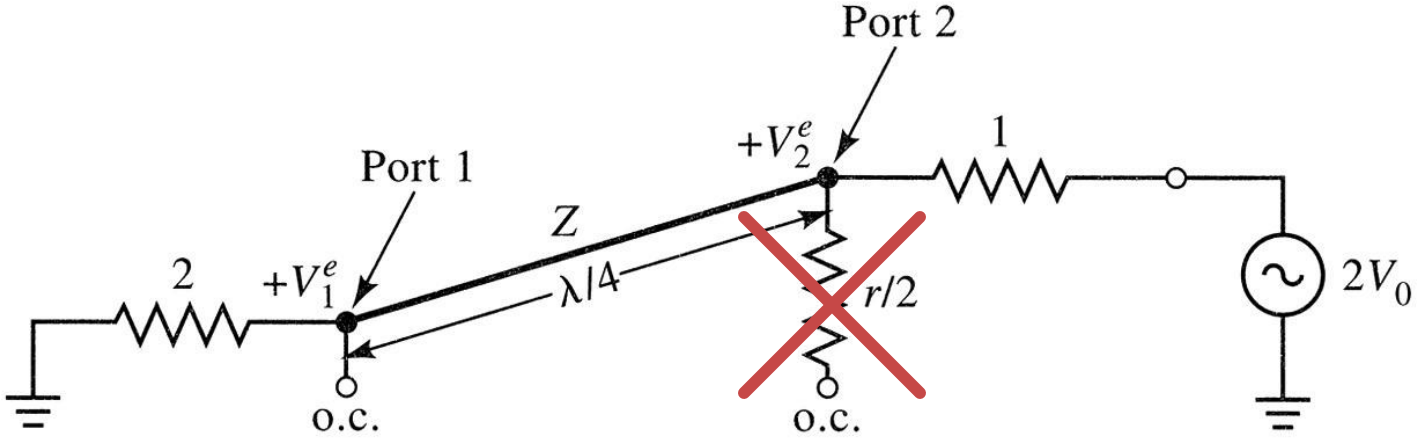
(a)



(b)

The Wilkinson power divider

- even mode, symmetry plane is open circuit



looking into port 2, λ/4 transformer with 2 load $Z_{in2}^e = \frac{Z^2}{2}$ if $Z = \sqrt{2}$ port 2 is matched $Z_{in2}^e = 1$

$$V(x) = V^+ \cdot \left(e^{-j\beta \cdot x} + \Gamma \cdot e^{j\beta \cdot x}\right)$$

x=0 at port 1

x=-λ/4 at port 2

$$V_2^e = V(-\lambda/4) = jV^+ \cdot (1-\Gamma) = V_0 \qquad V_1^e = V(0) = V^+ \cdot (1+\Gamma) = jV_0 \cdot \frac{\Gamma+1}{\Gamma-1}$$

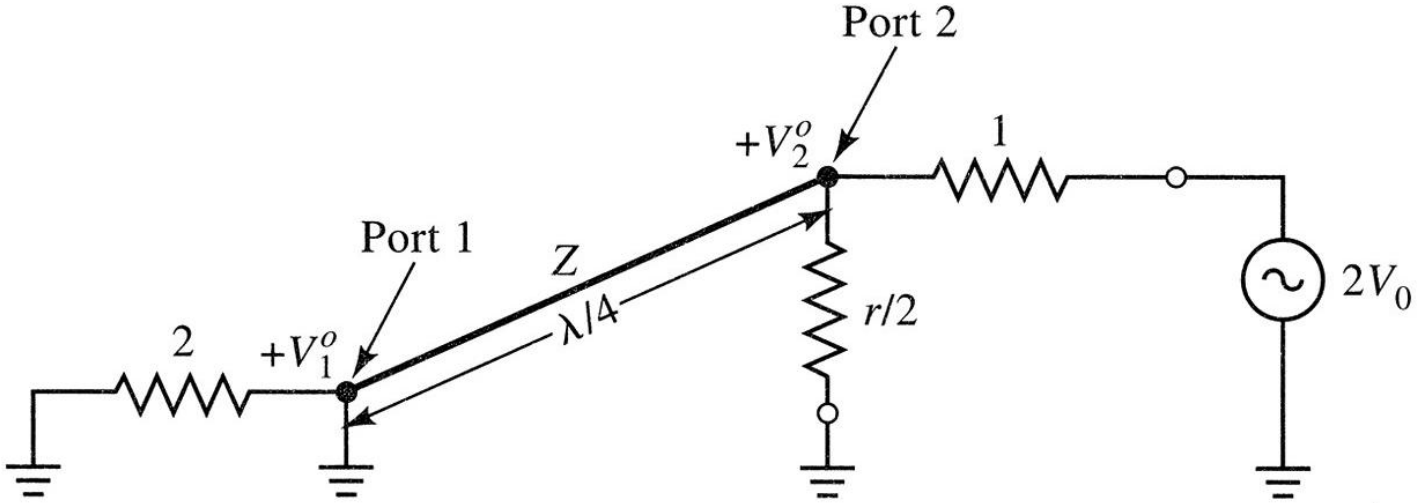
$Z_{in2}^e = 1$

Γ: reflection coefficient seen at port 1 looking toward the resistor of normalized value 2 from the transformer $Z = \sqrt{2}$

$$\Gamma = \frac{2-\sqrt{2}}{2+\sqrt{2}} \qquad V_1^e = -jV_0\sqrt{2}$$

The Wilkinson power divider

- odd mode, symmetry plane is grounded



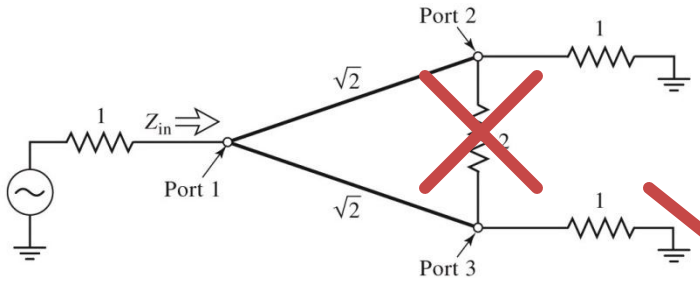
looking from port 2 the λ/4 line is short-circuited, impedance seen from port 2 is ∞ $Z_{in2}^o = r/2$ if $r = 2$ port 2 is matched

$Z_{in2}^o = 1 \rightarrow V_2^o = V_0$

$V_1^o = 0$ in the odd mode all the power is dissipated in the *r*/2 resistor

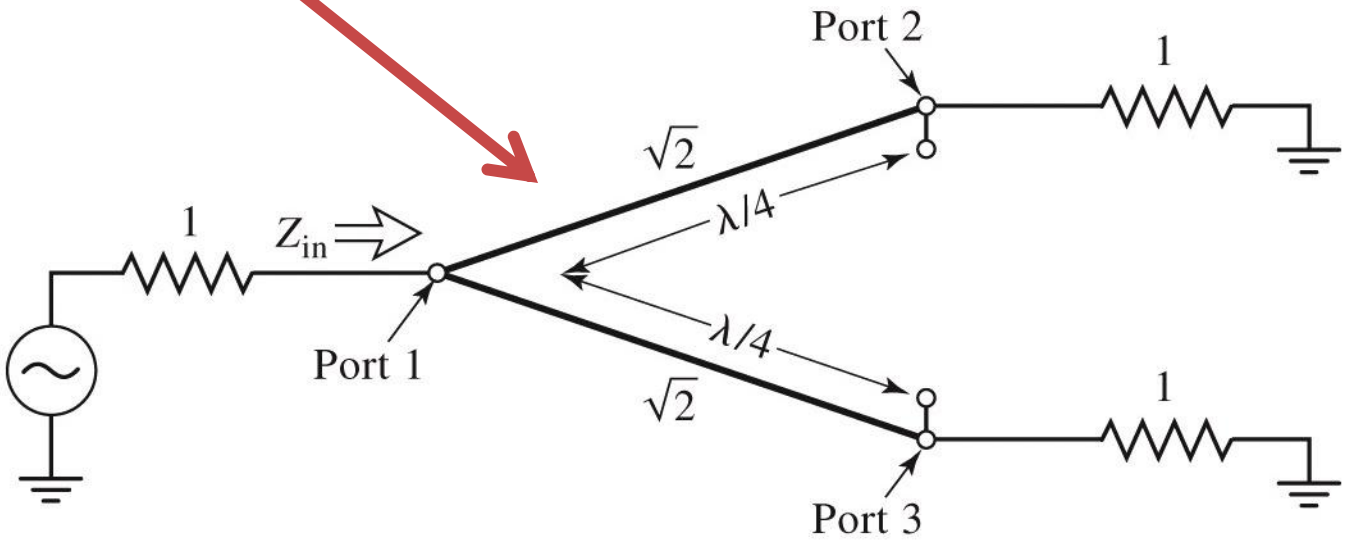
The Wilkinson power divider

- input impedance in port 1



two λ/4 transformers with load 1 in parallel

$$Z_{in1} = \frac{1}{2}\left(\sqrt{2}\right)^2 = 1$$



The Wilkinson power divider

- S parameters

$$Z_{in1} = \frac{1}{2}\left(\sqrt{2}\right)^2 = 1 \qquad S_{11} = 0$$

$$Z^e_{in2} = 1 \quad Z^o_{in2} = 1 \quad \text{and} \quad Z^e_{in3} = 1 \quad Z^o_{in3} = 1 \qquad S_{22} = S_{33} = 0$$

$$S_{12} = S_{21} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o} = -\frac{j}{\sqrt{2}}$$

and $S_{13} = S_{31} = -\frac{j}{\sqrt{2}}$

$S_{23} = S_{32} = 0$ due to short or open at bisection, both eliminate transfer between the ports + reciprocal circuit

The Wilkinson power divider

- at design frequency (length of the transformer equal to $\lambda_0/4$) we have **isolation** between the two output ports

$$[S] = \begin{bmatrix} 0 & -\frac{j}{\sqrt{2}} & -\frac{j}{\sqrt{2}} \\ -\frac{j}{\sqrt{2}} & 0 & 0 \\ -\frac{j}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

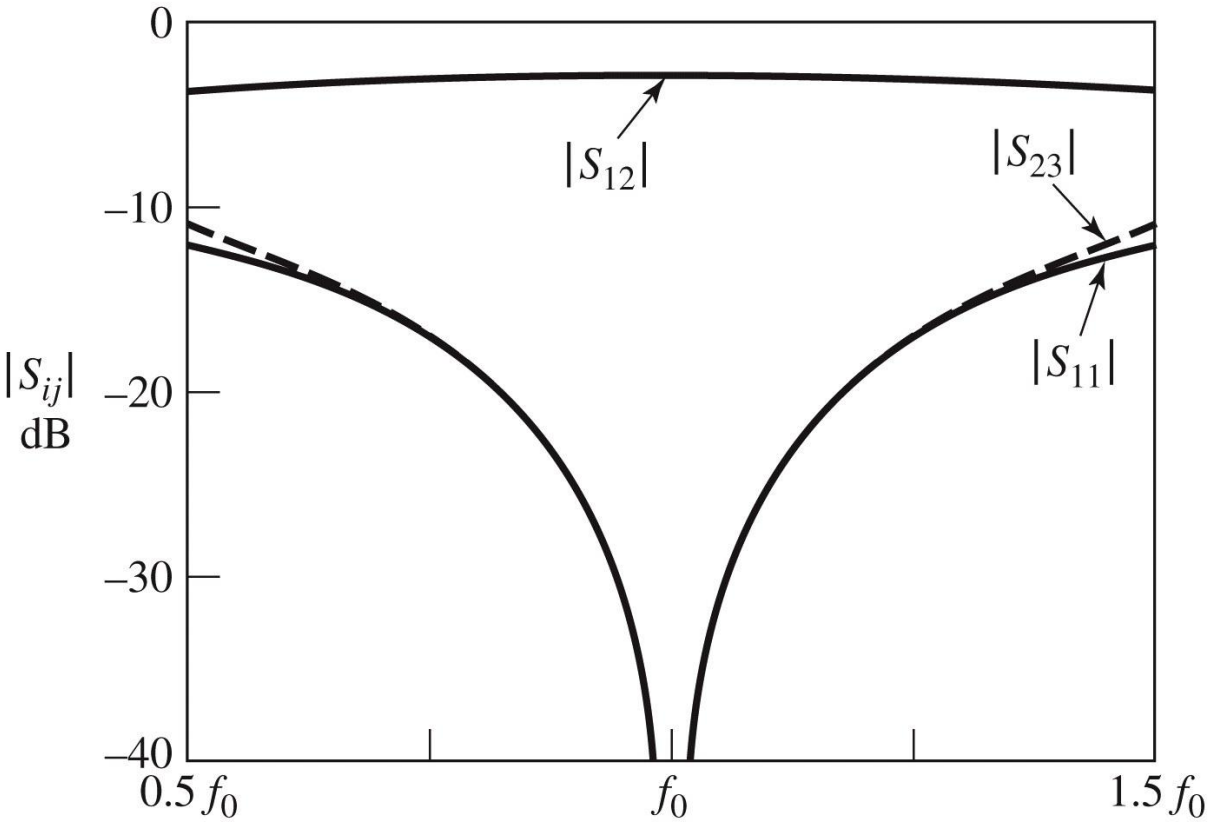


Figure 7.12
© John Wiley & Sons, Inc. All rights reserved.

The Wilkinson power divider

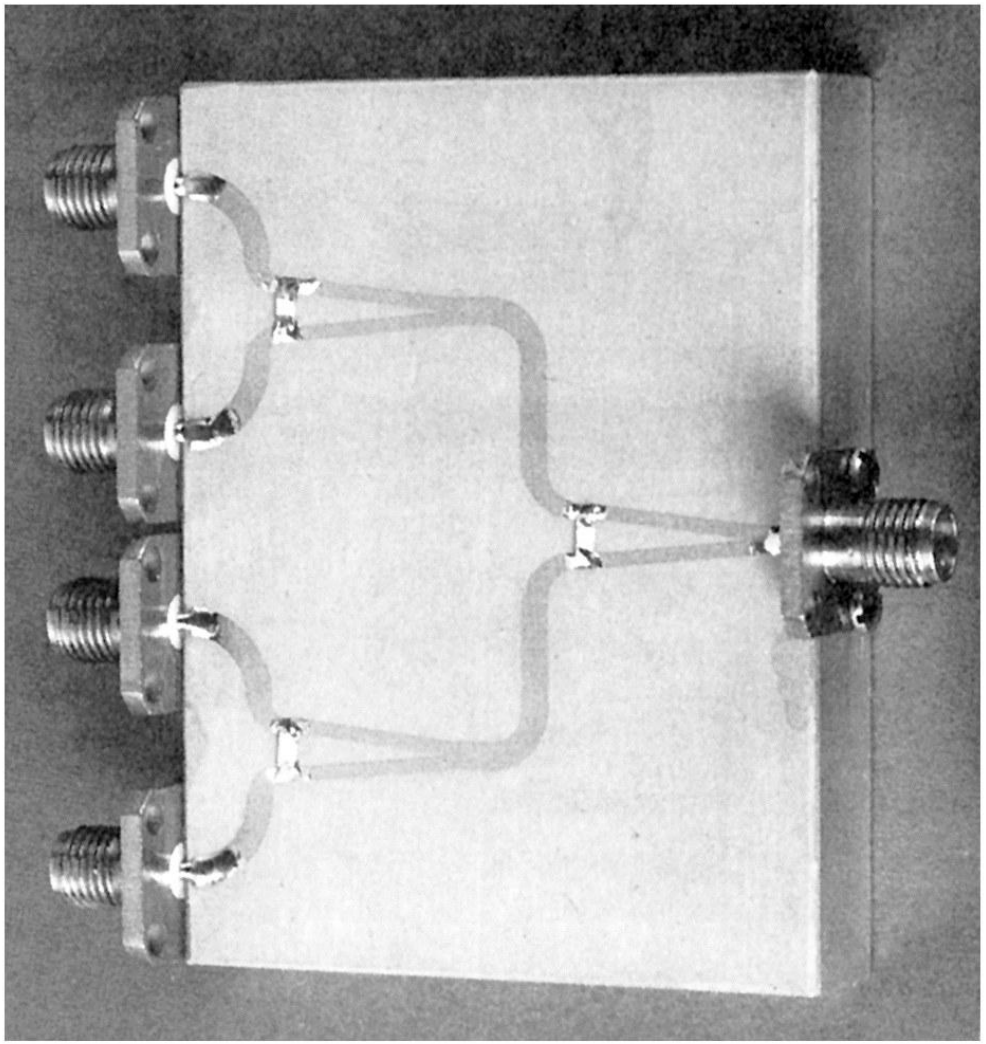
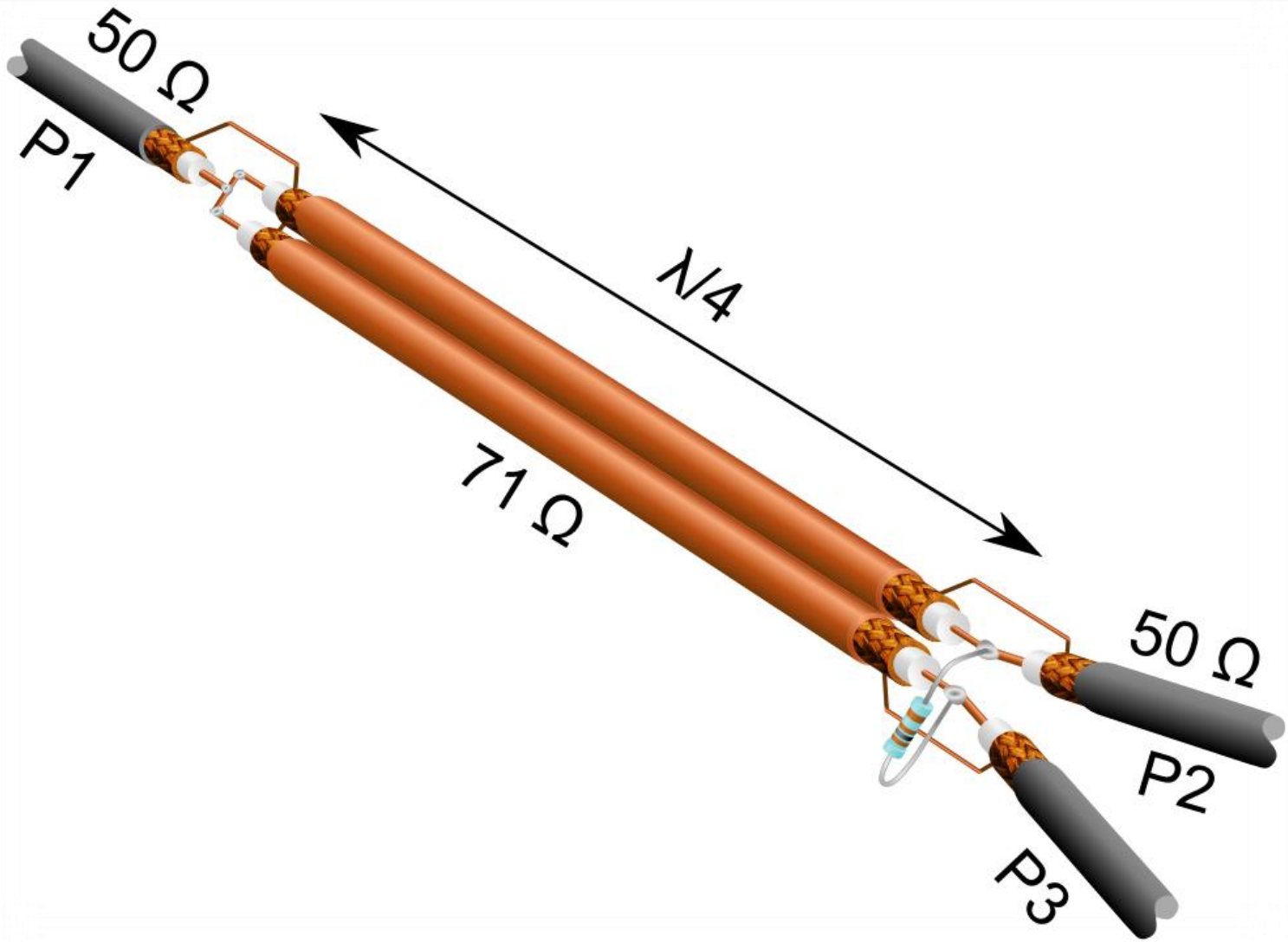


Figure 7.15
Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

- 3 X Wilkinson = 4-way power divider

The Wilkinson power divider



Directional couplers

Four-Port Networks

- A four-port network simultaneously:
  - matched at all ports
  - reciprocal
  - lossless
- is **always directional**
  - the signal power injected into one port is transmitted **only towards two** of the other three ports

$$[S]=\begin{bmatrix} 0 & \alpha & \beta\cdot e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta\cdot e^{j\phi} \\ \beta\cdot e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta\cdot e^{j\phi} & \alpha & 0 \end{bmatrix}$$

Directional Coupler

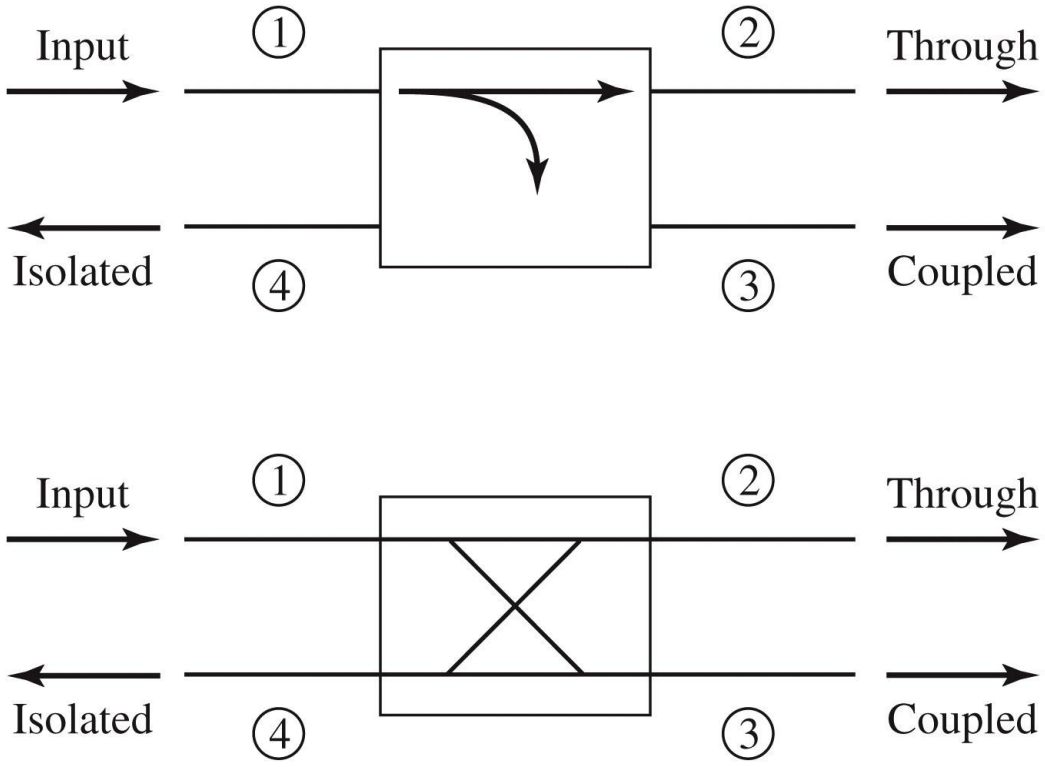


Figure 7.4
© John Wiley & Sons, Inc. All rights reserved.

$$|S_{12}|^2 = \alpha^2 = 1 - \beta^2$$

$$|S_{13}|^2 = \beta^2$$

Coupling

$$C = 10\log\frac{P_1}{P_3} = -20\cdot\log(\beta)\,[\text{dB}]$$

Directivity

$$D = 10\log\frac{P_3}{P_4} = 20\cdot\log\left(\frac{\beta}{|S_{14}|}\right)[\text{dB}]$$

Isolation

$$I = 10\log\frac{P_1}{P_4} = -20\cdot\log|S_{14}|\,[\text{dB}]$$

$$I = D + C\,,\ [dB]$$

Four-Port Networks

- two particular choices commonly occur in practice
  - A Symmetric Coupler $\theta = \phi = \pi/2$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

  - An Antisymmetric Coupler $\theta = 0, \phi = \pi$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Hybrid Couplers

Hybrid Couplers are directional couplers with 3 dB coupling factor

$$\alpha = \beta = 1/\sqrt{2}$$

The cuadrature (90°) hybrid

$$(\theta = \phi = \pi/2)$$

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

The 180° ring hybrid (rat-race)

$$(\theta = 0\,, \phi = \pi)$$

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

The cuadrature (90°) hybrid

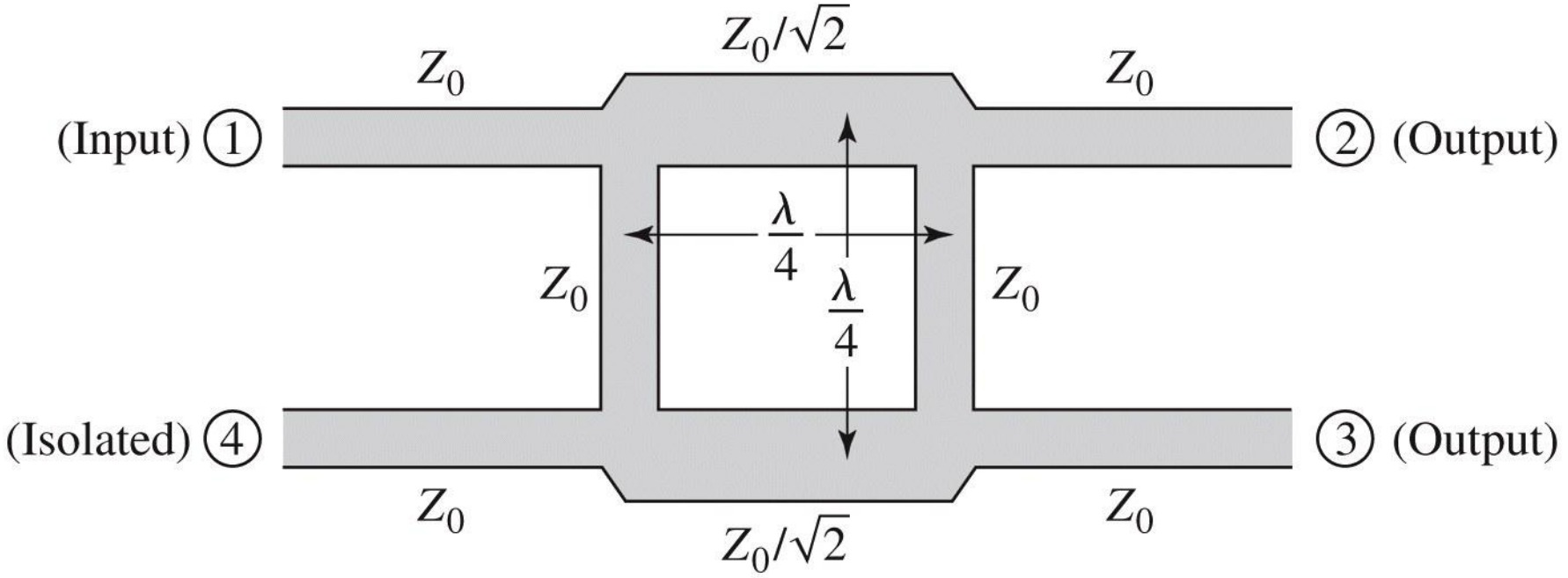


Figure 7.21
© John Wiley & Sons, Inc. All rights reserved.

$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

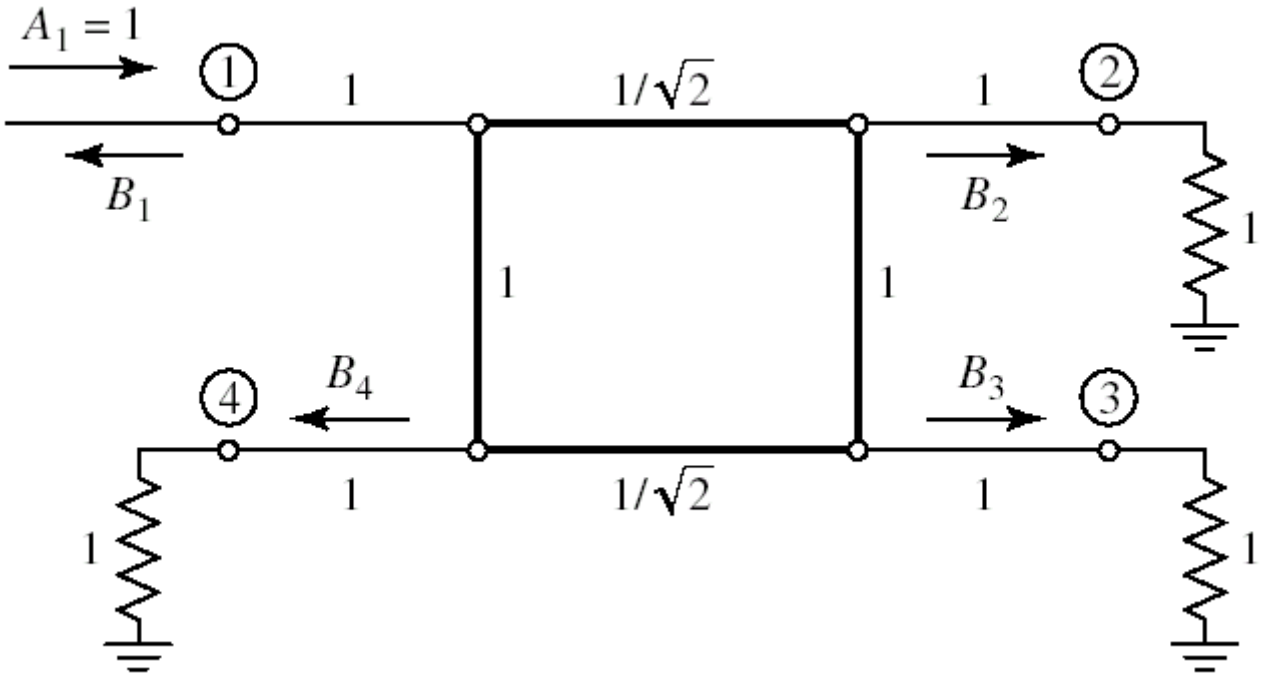
Even/Odd Mode Analysis

- In linear circuits we can use the superposition principle
- advantages
  - reduction of the circuit complexity
  - decrease of the number of ports (**main** advantage)

Response (ODD + EVEN) = Response (ODD) + Response (EVEN)

We can benefit from existing symmetries !!

Even/Odd Mode Analysis



Even/Odd Mode Analysis

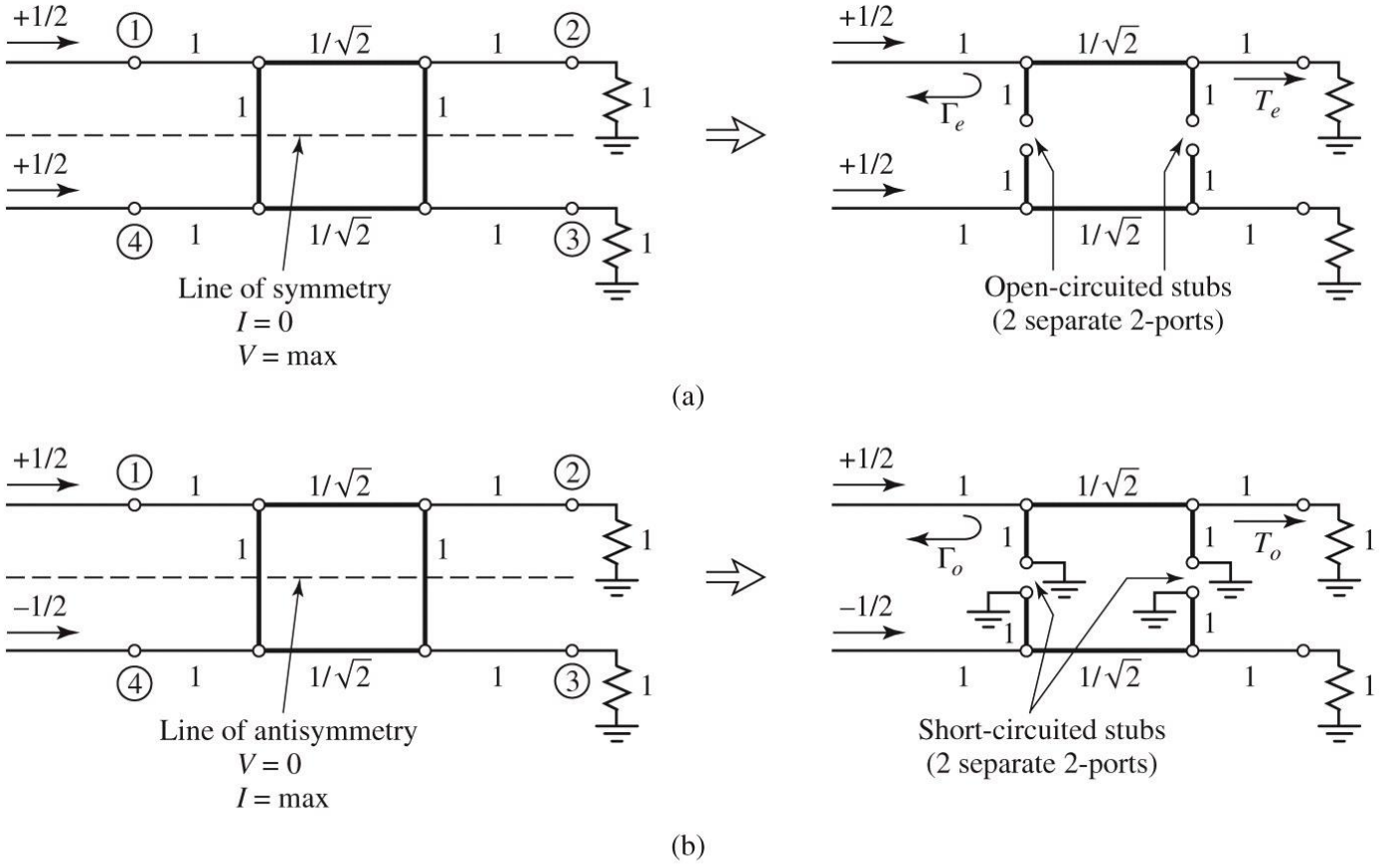


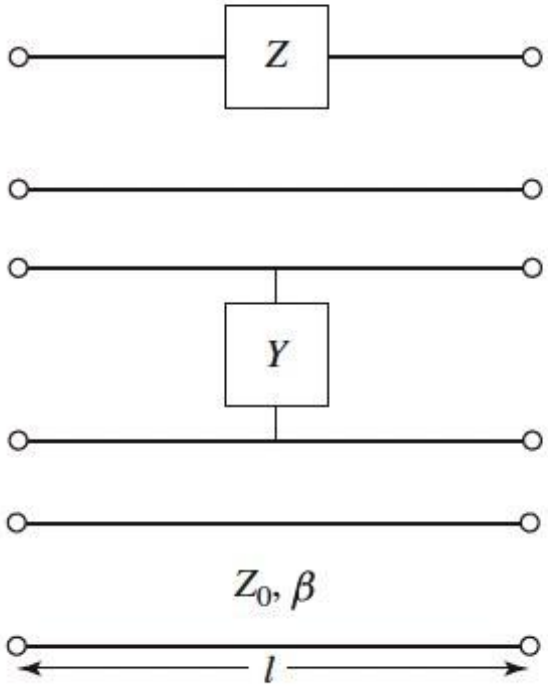
Figure 7.23

© John Wiley & Sons, Inc. All rights reserved.

$$b_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \qquad b_2 = \frac{1}{2}T_e + \frac{1}{2}T_o \qquad b_3 = \frac{1}{2}T_e - \frac{1}{2}T_o \qquad b_4 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$$

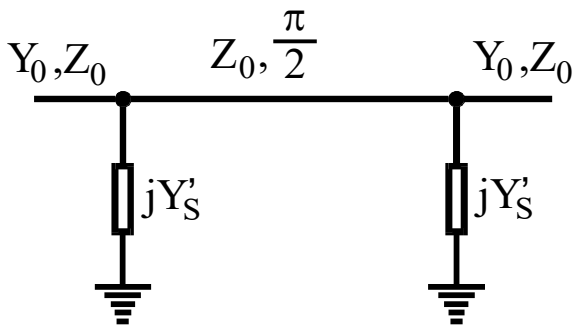
Library of ABCD matrices

TABLE 4.1 *ABCD* Parameters of Some Useful Two-Port Circuits

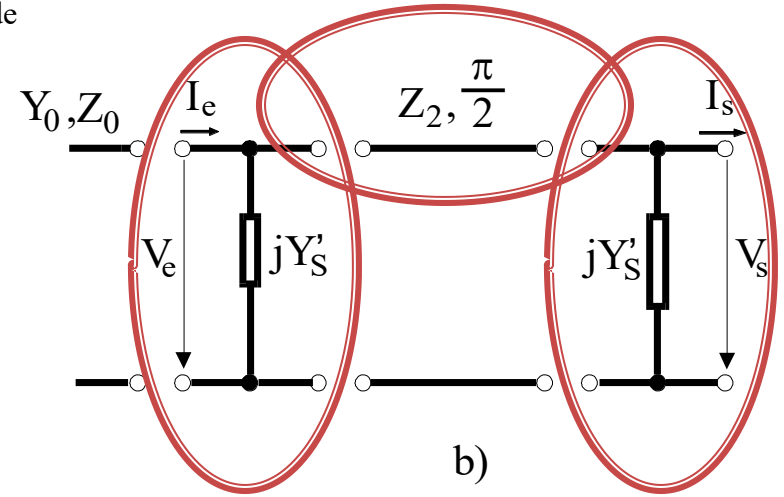
Circuit	*ABCD* Parameters
	$A = 1$ $B = Z$ $C = 0$ $D = 1$
	$A = 1$ $B = 0$ $C = Y$ $D = 1$
	$A = \cos \beta\ell$ $B = jZ_0 \sin \beta\ell$ $C = jY_0 \sin \beta\ell$ $D = \cos \beta\ell$

S parameters (from ABCD)

$$Y'_S = \begin{cases} Y_1 & \text{even mode} \\ -Y_1 & \text{odd mode} \end{cases}$$



a)



b)

$$\begin{bmatrix} V_e \\ I_e \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ jY'_S & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & jZ_2 \\ jY_2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ jY'_S & 1 \end{bmatrix} \cdot \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

$$\begin{bmatrix} V_e \\ I_e \end{bmatrix} = \begin{bmatrix} -Y'_S Z_2 & jZ_2 \\ -jY'^2_S Z_2 + jY_2 & -Y'_S Z_2 \end{bmatrix} \cdot \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

$$S_{11} = \frac{j\frac{Z_2}{Z_0} - Z_0\left(-jY'^2_S Z_2 + jY_2\right)}{-2Y'_S Z_2 + j\frac{Z_2}{Z_0} + Z_0\left(-jY'^2_S + jY_2\right)} \qquad S_{12} = \frac{2\left[(-Y'_S Z_2)^2 - jZ_2\left(-jY'^2_S Z_2 + jY_2\right)\right]}{-2Y'_S Z_2 + j\frac{Z_2}{Z_0} + Z_0\left(-jY'^2_S Z_2 + jY_2\right)}$$

$$\Gamma = S_{11} = \frac{j\left(z_2 - y_2 + y'^2_S z_2\right)}{-2y'_S z_2 + j\left(z_2 + y_2 - y'^2_S z_2\right)} = S_{22}$$

$$S_{21} = \frac{2}{-2Y'_S Z_2 + j\frac{Z_2}{Z_0} + Z_0\left(-jY'^2_S Z_2 + jY_2\right)} \qquad S_{22} = \frac{j\frac{Z_2}{Z_0} - Z_0\left(-jY'^2_S Z_2 + jY_2\right)}{-2Y'_S Z_2 + j\frac{Z_2}{Z_0} + Z_0\left(-jY'^2_S Z_2 + jY_2\right)}$$

$$T = S_{21} = \frac{2}{-2y'_S z_2 + j\left(z_2 + y_2 - y'^2_S z_2\right)} = S_{12}$$

Relation between two port S parameters and ABCD parameters

$$A = \sqrt{\frac{Z_{01}}{Z_{02}}}\,\frac{(1+S_{11}-S_{22}-\Delta S)}{2S_{21}}$$

$$B = \sqrt{Z_{01}Z_{02}}\,\frac{(1+S_{11}+S_{22}+\Delta S)}{2S_{21}}$$

$$C = \frac{1}{\sqrt{Z_{01}Z_{02}}}\,\frac{1-S_{11}-S_{22}+\Delta S}{2S_{21}}$$

$$D = \sqrt{\frac{Z_{02}}{Z_{01}}}\,\frac{1-S_{11}+S_{22}-\Delta S}{2S_{21}}$$

$$\Delta S = S_{11}S_{22} - S_{12}S_{21}$$

$$S_{11} = \frac{AZ_{02}+B-CZ_{01}Z_{02}-DZ_{01}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$$

$$S_{12} = \frac{2(AD-BC)\sqrt{Z_{01}Z_{02}}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$$

$$S_{21} = \frac{2\sqrt{Z_{01}Z_{02}}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$$

$$S_{22} = \frac{-AZ_{02}+B-CZ_{01}Z_{02}+DZ_{01}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$$

Matching and coupling factor

$$\Gamma_e = \frac{j\cdot\left(z_2 - y_2 + y_1^2 z_2\right)}{-2y_1 z_2 + j\left(z_2 + y_2 - y_1^2 z_2\right)}$$

$$\Gamma_o = \frac{j\cdot\left(z_2 - y_2 + y_1^2 z_2\right)}{2y_1 z_2 + j\left(z_2 + y_2 - y_1^2 z_2\right)}$$

$$T_e = \frac{2}{-2y_1 z_2 + j\cdot\left(z_2 + y_2 - y_1^2 z_2\right)}$$

$$T_o = \frac{2}{2y_1 z_2 + j\cdot\left(z_2 + y_2 - y_1^2 z_2\right)}$$

$$b_1 = \frac{\Gamma_e + \Gamma_o}{2} = \frac{z_2^2 - \left(y_2 - y_1^2 z_2\right)^2}{(2y_1 z_2)^2 + \left(z_2 + y_2 - y_1^2 z_2\right)^2}$$

$$b_2 = \frac{T_e + T_o}{2} = \frac{-2j\left(z_2 + y_2 - y_1^2 z_2\right)}{(2y_1 z_2)^2 + \left(z_2 + y_2 - y_1^2 z_2\right)^2}$$

$$b_3 = \frac{T_e - T_o}{2} = \frac{-4y_1 z_2}{(2y_1 z_2)^2 + \left(z_2 + y_2 - y_1^2 z_2\right)^2}$$

$$b_4 = \frac{\Gamma_e - \Gamma_o}{2} = \frac{-2jy_1 z_2\left(z_2 - y_2 + y_1^2 z_2\right)}{(2y_1 z_2)^2 + \left(z_2 + y_2 - y_1^2 z_2\right)^2}$$

$$b_1 = 0 \Rightarrow z_2 - y_2 + y_1^2 z_2 = 0 \Rightarrow z_2^2 = \frac{1}{1 + y_1^2}$$

$$\boxed{y_2^2 = 1 + y_1^2}$$

$$C = 10\log\frac{P_1}{P_3} = -20\log|b_3|\ ,dB$$

$$b_1 = 0\ b_4 = 0\ b_3 = -y_1 z_2\ \ b_2 = -jz_2$$

$$b_3 = -\frac{\sqrt{y_2^2 - 1}}{y_2}\ ,\ b_2 = -\frac{j}{y_2}$$

$$\boxed{\beta = \frac{\sqrt{y_2^2 - 1}}{y_2}}$$

$$b_3 = -C$$

$$b_2 = -j\sqrt{1 - C^2}$$

$$[S] = \begin{bmatrix} 0 & -j\sqrt{1-C^2} & -C & 0 \\ -j\sqrt{1-C^2} & 0 & 0 & -C \\ -C & 0 & 0 & -j\sqrt{1-C^2} \\ 0 & -C & -j\sqrt{1-C^2} & 0 \end{bmatrix}$$

The cuadrature (90°) hybrid

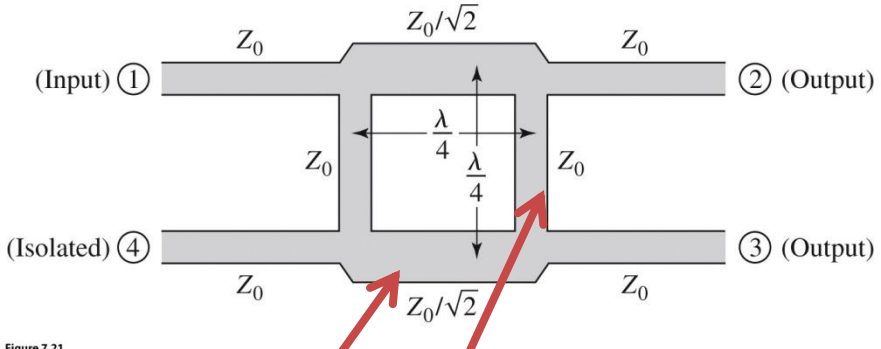


Figure 7.21
© John Wiley & Sons, Inc. All rights reserved.

$$y_2^2 = 1 + y_1^2$$

$$|\beta| = \frac{\sqrt{y_2^2 - 1}}{y_2}$$

$$C[\text{dB}] = -20 \cdot \log_{10} \frac{\sqrt{y_2^2 - 1}}{y_2}$$

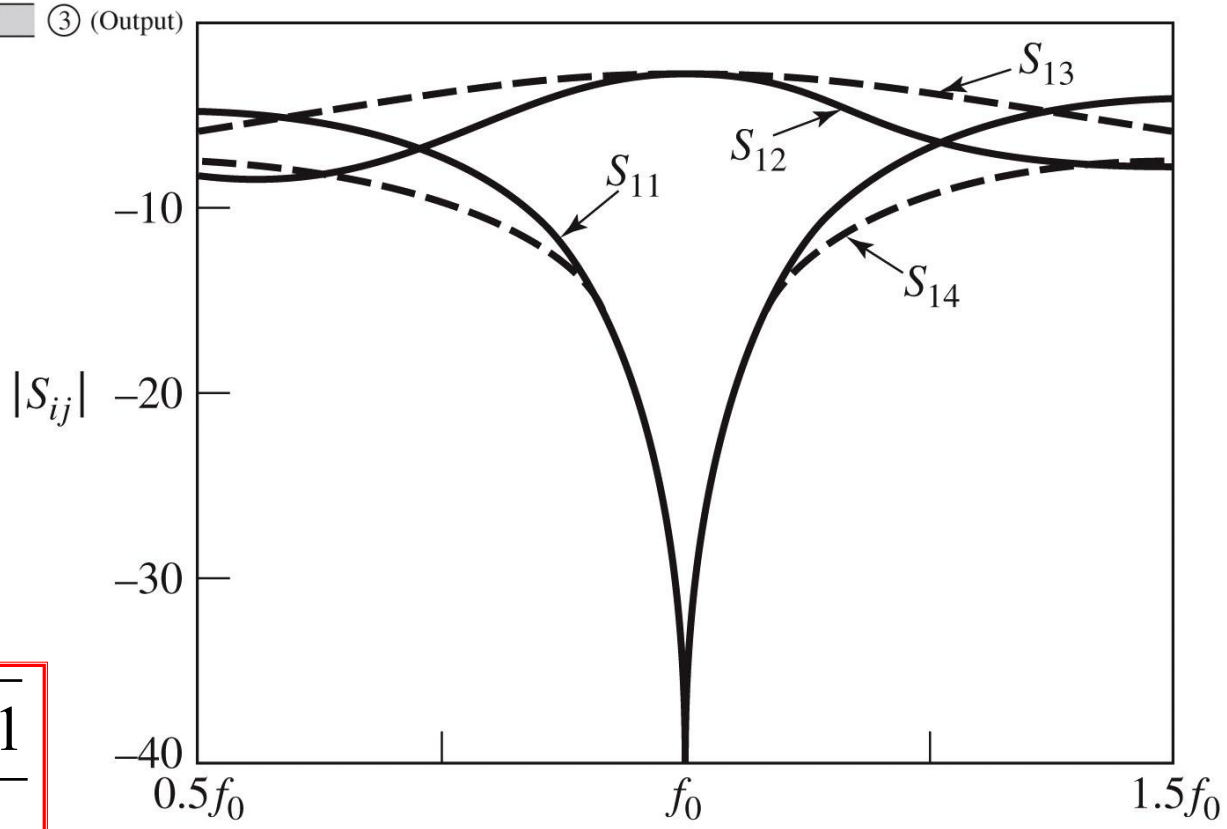


Figure 7.25
© John Wiley & Sons, Inc. All rights reserved.

Example

Design a cuadrature (90°) hybrid working on 50 Ω, and plot the S parameters between

$$0.5 f_0 \text{ and } 1.5 f_0, \text{ where } f_0$$

is the frequency at which the length of the branches is $\lambda/4$

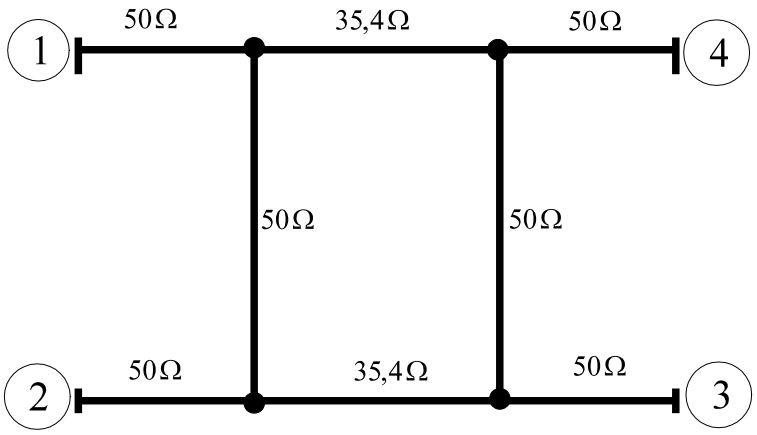
Solution

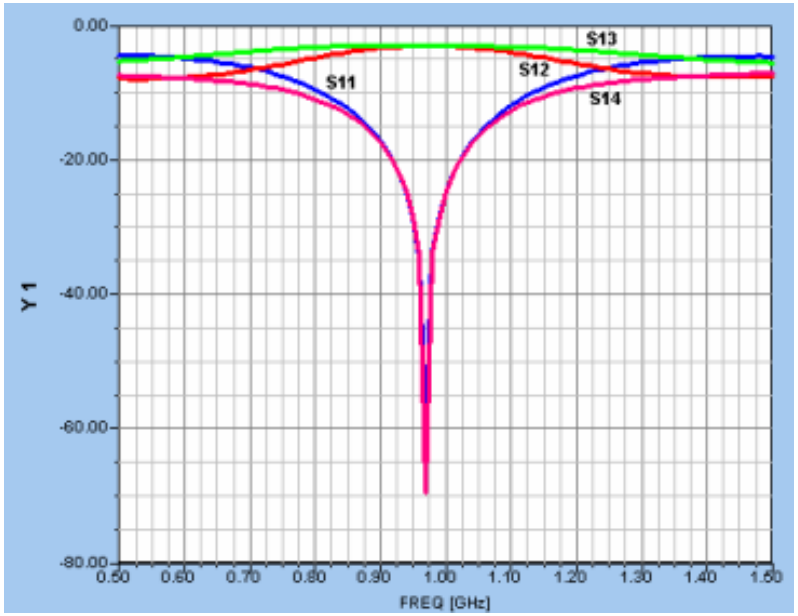
A cuadrature (90°) hybrid has C = 3dB, then $\beta = 1/\sqrt{2}$

$$y_2 = \sqrt{2} \quad \text{and} \quad y_1 = 1$$

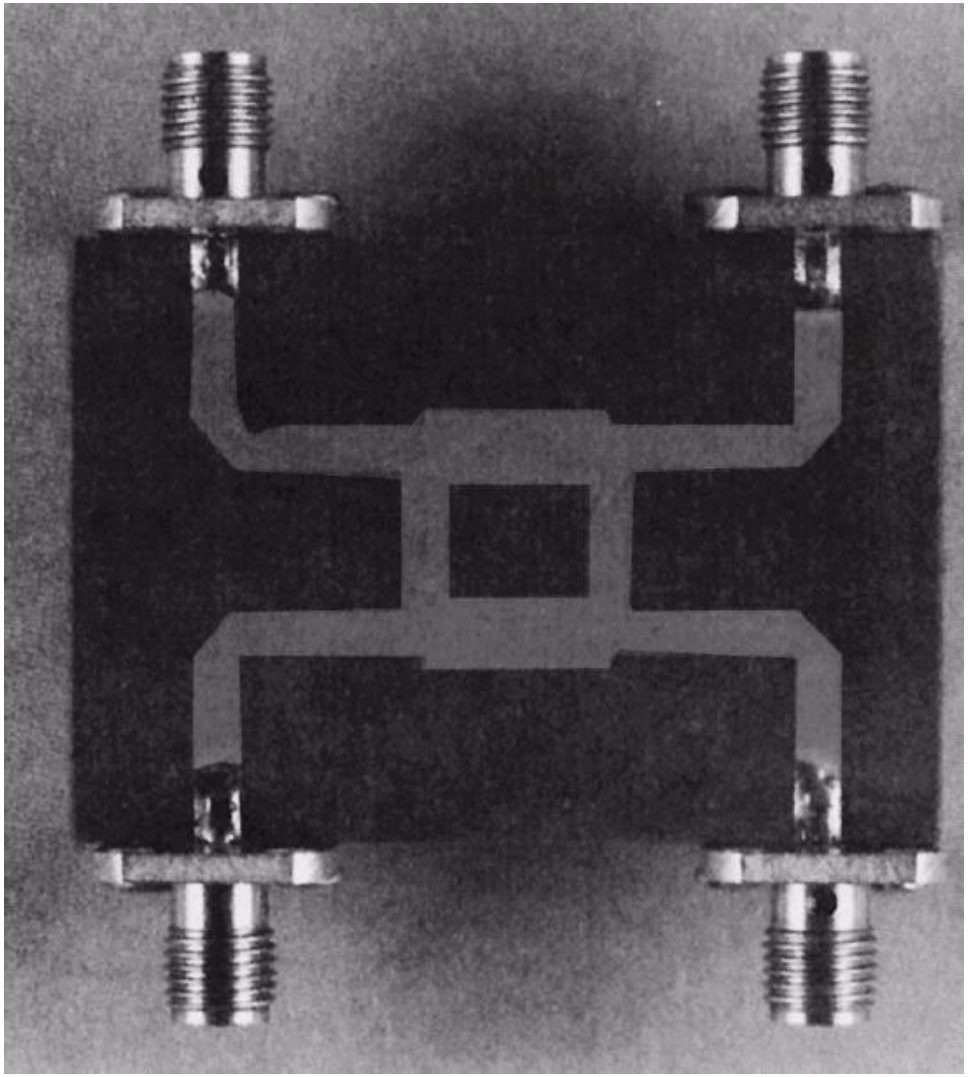
$Z_0 = 50\Omega$ the characteristic impedances will be:

$$Z_1 = Z_0 = 50\Omega \quad Z_2 = \frac{Z_0}{\sqrt{2}} = 35.4\Omega$$





The cuadrature (90°) hybrid



The cuadrature (90°) hybrid

- eight-way microstrip power divider with six quadrature hybrids in a Bailey configuration

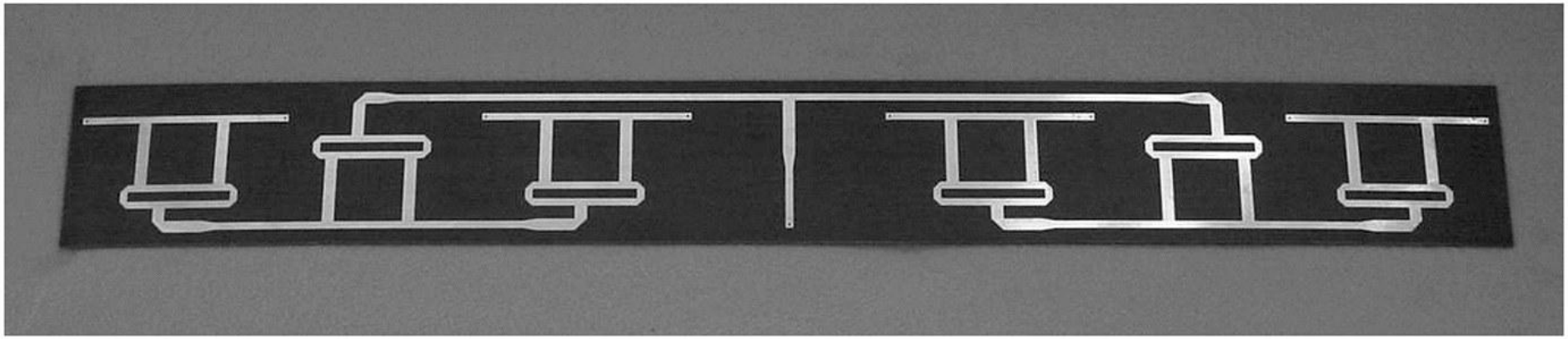
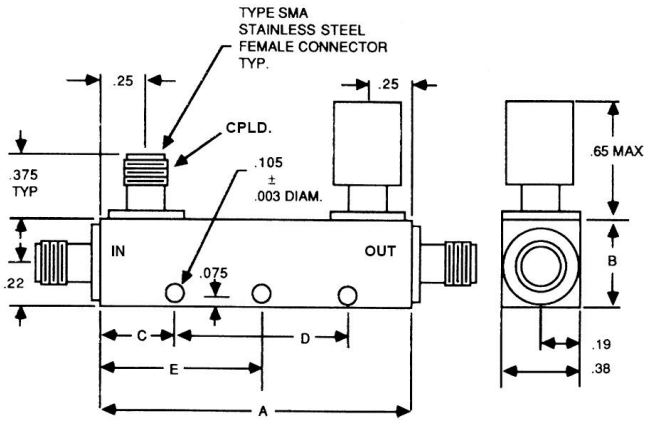


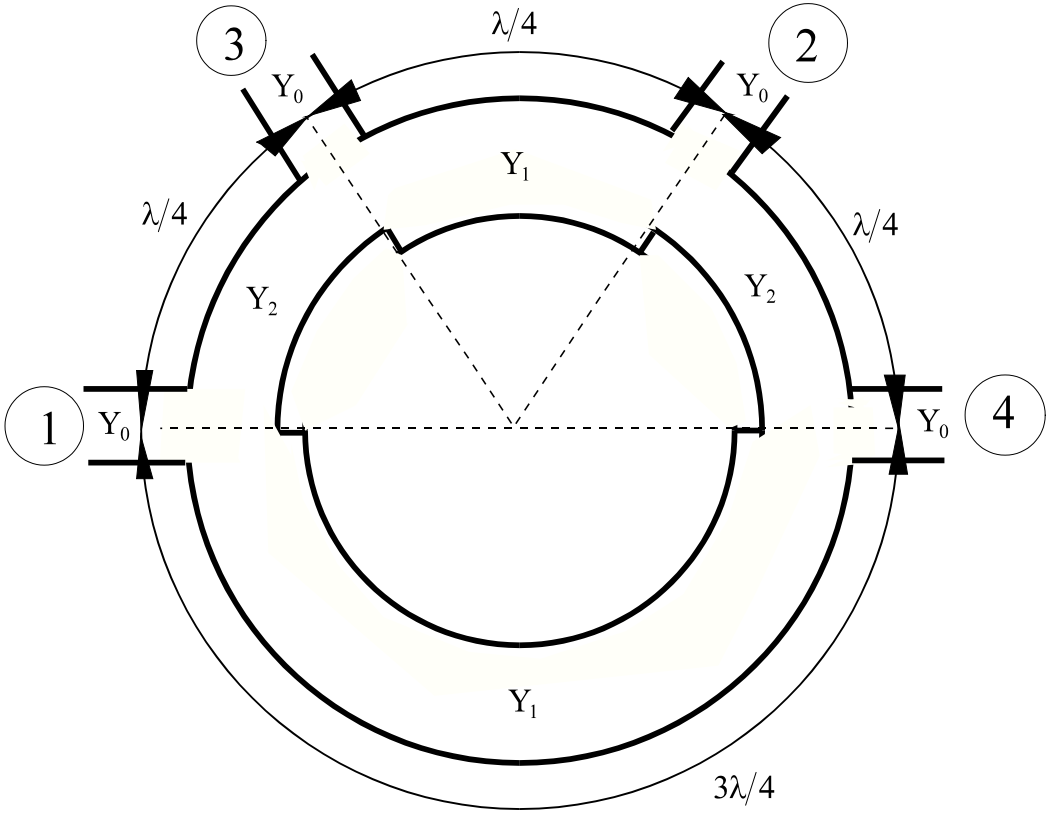
Figure 7.24
Courtesy of ProSensing, Inc., Amherst, Mass.

Datasheet



Model No.	Frequency Range (Ghz)	Coupling † (dB)	Freq. Sens. (dB)	Insertion Loss (dB)		Directivity (dB min.)	VSWR max.	
				Excl. Cpld Pwr	True		Primary Line	Secondary Line
MDC6223-6	0.5-1.0	6 ±1.00	±0.60	0.20	1.80	25	1.15	1.15
MDC6223-10	0.5-1.0	10 ±1.25	±0.75	0.20	0.80	25	1.10	1.10
MDC6223-20	0.5-1.0	20 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6223-30	0.5-1.0	30 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6224-6	1.0-2.0	6 ±1.00	±0.60	0.20	1.80	25	1.15	1.15
MDC6224-10	1.0-2.0	10 ±1.25	±0.75	0.20	0.80	25	1.10	1.10
MDC6224-20	1.0-2.0	20 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6224-30	1.0-2.0	30 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6225-6	2.0-4.0	6 ±1.00	±0.60	0.20	1.80	22	1.15	1.15
MDC6225-10	2.0-4.0	10 ±1.25	±0.75	0.20	0.80	22	1.15	1.15
MDC6225-20	2.0-4.0	20 ±1.25	±0.75	0.15	0.20	22	1.15	1.15
MDC6225-30	2.0-4.0	30 ±1.25	±0.75	0.15	0.20	22	1.15	1.15
MDC6266-6	2.6-5.2	6 ±1.00	±0.60	0.20	1.80	20	1.25	1.25
MDC6266-10	2.6-5.2	10 ±1.25	±0.75	0.20	0.80	20	1.25	1.25
MDC6266-20	2.6-5.2	20 ±1.25	±0.75	0.20	0.25	20	1.25	1.25
MDC6266-30	2.6-5.2	30 ±1.25	±0.75	0.20	0.20	20	1.25	1.25
MDC6226-6	4.0-8.0	6 ±1.00	±0.60	0.25	1.90	20	1.25	1.25
MDC6226-10	4.0-8.0	10 ±1.25	±0.75	0.25	0.90	20	1.25	1.25
MDC6226-20	4.0-8.0	20 ±1.25	±0.75	0.25	0.30	20	1.25	1.25
MDC6226-30	4.0-8.0	30 ±1.25	±0.75	0.25	0.25	20	1.25	1.25
MDC6227-6	7.0-12.4	6 ±1.00	±0.50	0.30	2.00	17	1.30	1.30
MDC6227-10	7.0-12.4	10 ±1.00	±0.50	0.30	1.00	17	1.30	1.30
MDC6227-20	7.0-12.4	20 ±1.00	±0.50	0.30	0.35	17	1.30	1.30

The 180° ring hybrid (rat-race)



The 180° ring hybrid

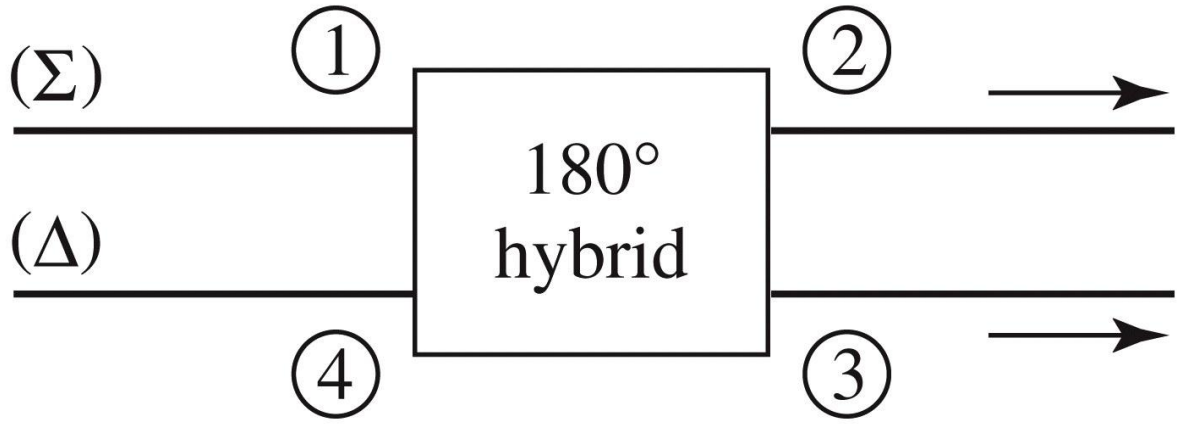


Figure 7.41
© John Wiley & Sons, Inc. All rights reserved.

- The 180° ring hybrid can be operated in different modes:
  - a signal applied to port 1 will be evenly split into two in-phase components at ports 2 and 3
  - input applied to port 4 it will be equally split into two components with a 180◦ phase difference at ports 2 and 3
  - input signals applied at ports 2 and 3, the sum of the inputs will be formed at port 1, while the difference will be formed at port 4 (power combiner)

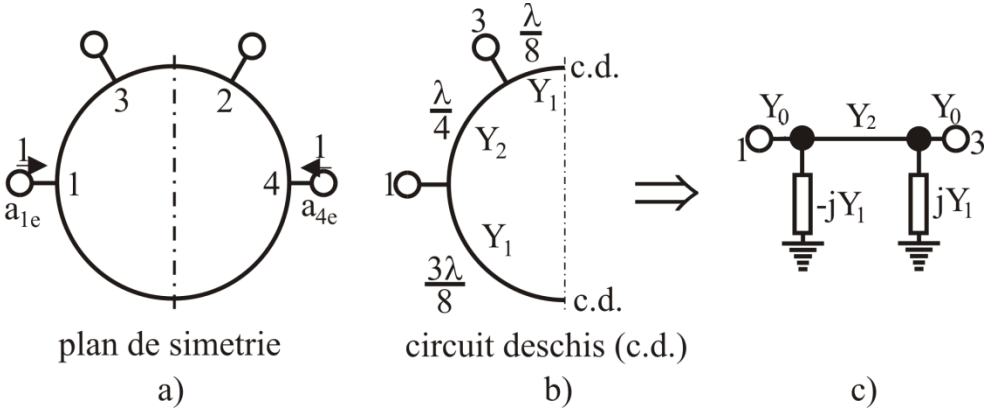
Even/Odd Mode Analysis

- In linear circuits we can use the superposition principle
- advantages
  - reduction of the circuit complexity
  - decrease of the number of ports (**main** advantage)

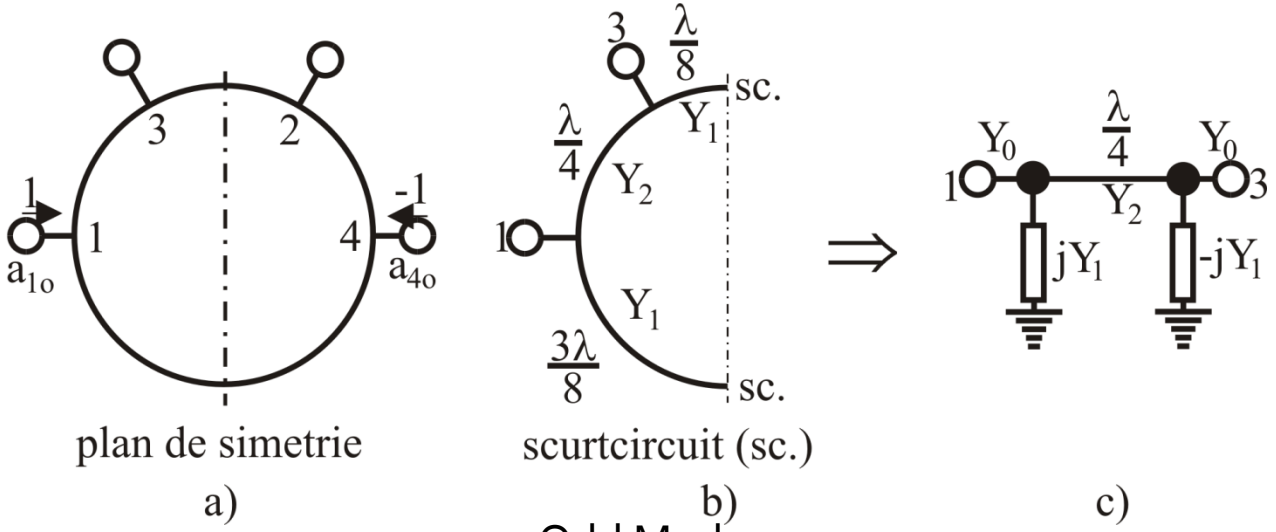
Response (ODD + EVEN) = Response (ODD) + Response (EVEN)

We can benefit from existing symmetries !!

Even/Odd Mode Analysis



Even Mode

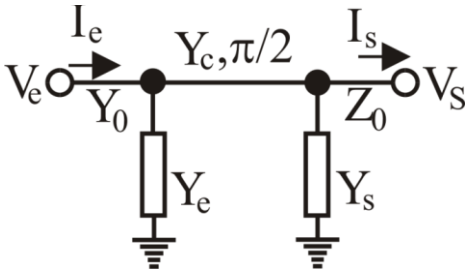


Odd Mode

Even/Odd Mode Analysis

$$S_{11} = \frac{jz_2y_s + jz_2 - j(y_2 + y_e y_s z_2) - jy_e z_2}{jz_2y_s + jz_2 + j(y_2 + y_e y_s z_2) + jy_e z_2}$$

$$S_{12} = \frac{2}{jz_2y_s + jz_2 + j(y_2 + y_e y_s z_2) + jy_e z_2}$$



$$S_{21} = \frac{2}{jz_2y_s + jz_2 + j(y_2 + y_e y_s z_2) + jy_e z_2}$$

$$S_{22} = \frac{-jz_2y_s + jz_2 - j(y_2 + y_e y_s z_2) + jy_e z_2}{jz_2y_s + jz_2 + j(y_2 + y_e y_s z_2) + jy_e z_2}$$

Even mode:

$y_e = -jy_1$

$y_s = jy_1$

Matching condition

$$y_1^2 + y_2^2 = 1$$

Odd mode:

$y_e = jy_1$

$y_s = -jy_1$

$$S_{11e} = \frac{z_2 - y_2 - y_1^2 z_2 + 2jz_2y_1}{z_2 + y_2 + y_1^2 z_2}$$

$$S_{12e} = S_{21e} = \frac{-2j}{z_2 + y_2 + y_1^2 z_2}$$

$$S_{22e} = \frac{z_2 - y_2 - y_1^2 z_2 - 2jz_2y_1}{z_2 + y_2 + y_1^2 z_2}$$

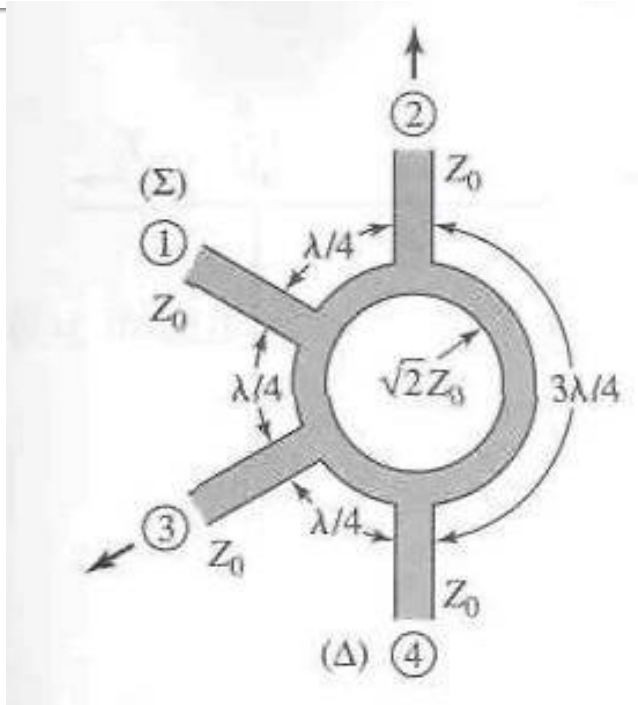
$$[S] = \begin{bmatrix} 0 & 0 & -jy_2 & jy_1 \\ 0 & 0 & -jy_1 & -jy_2 \\ -jy_2 & -jy_1 & 0 & 0 \\ jy_1 & -jy_2 & 0 & 0 \end{bmatrix}$$

$$S_{11o} = \frac{z_2 - y_2 - y_1^2 z_2 - 2jz_2y_1}{z_2 + y_2 + y_1^2 z_2}$$

$$S_{12o} = S_{21o} = \frac{-2j}{z_2 + y_2 + y_1^2 z_2}$$

$$S_{22o} = \frac{z_2 - y_2 - y_1^2 z_2 + 2jz_2y_1}{z_2 + y_2 + y_1^2 z_2}$$

The 180° ring hybrid



$$[S]=\begin{bmatrix} 0 & -jy_2 & -jy_1 & 0 \\ -jy_2 & 0 & 0 & jy_1 \\ -jy_1 & 0 & 0 & -jy_2 \\ 0 & jy_1 & -jy_2 & 0 \end{bmatrix} = -j\begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

$$C(dB) = -20\log(\beta) = -20\log(y_1)$$

Example

Design a ring (180°) hybrid working on 50 Ω, and plot the S parameters between 0.5 and 1.5 of the design frequency.

$$C\,[\mathrm{dB}] = -20\log(y_1)$$

$$\sqrt{2}Z_0 = 70.7\Omega$$

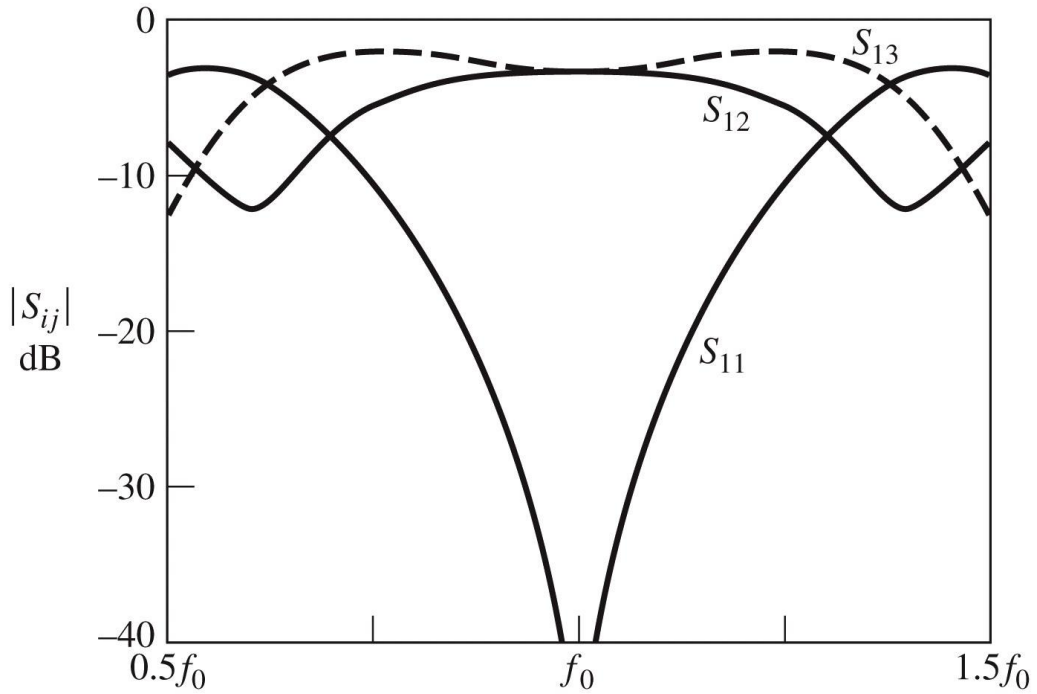
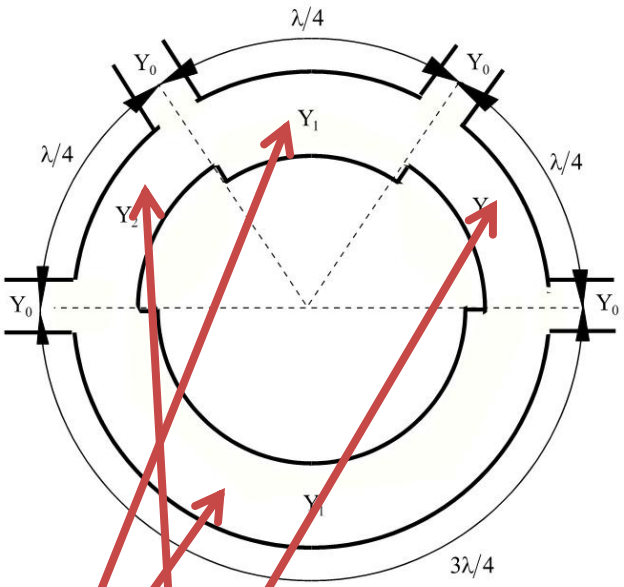


Figure 7.46
© John Wiley & Sons, Inc. All rights reserved.

The 180° ring hybrid



$$y_1^2 + y_2^2 = 1$$

$$|\beta| = y_1$$

$$C\,[\text{dB}] = -20 \cdot \log_{10}(y_1)$$

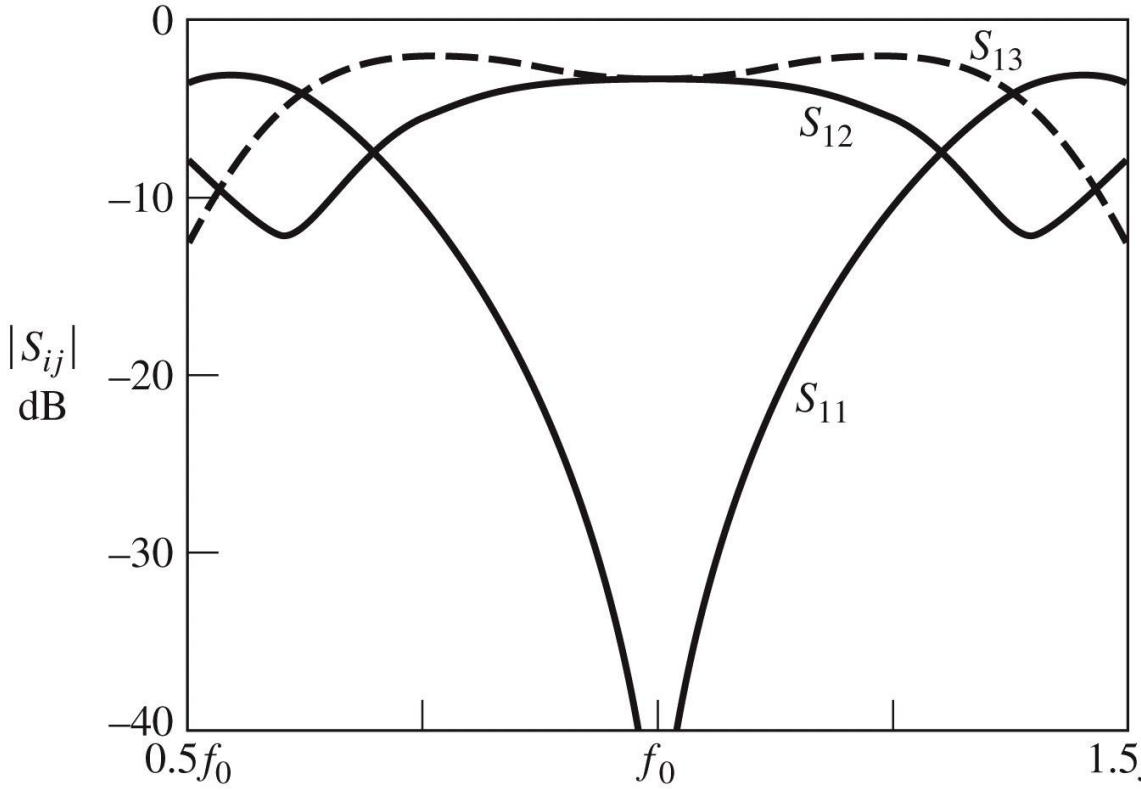


Figure 7.46
© John Wiley & Sons, Inc. All rights reserved.

The 180° ring hybrid

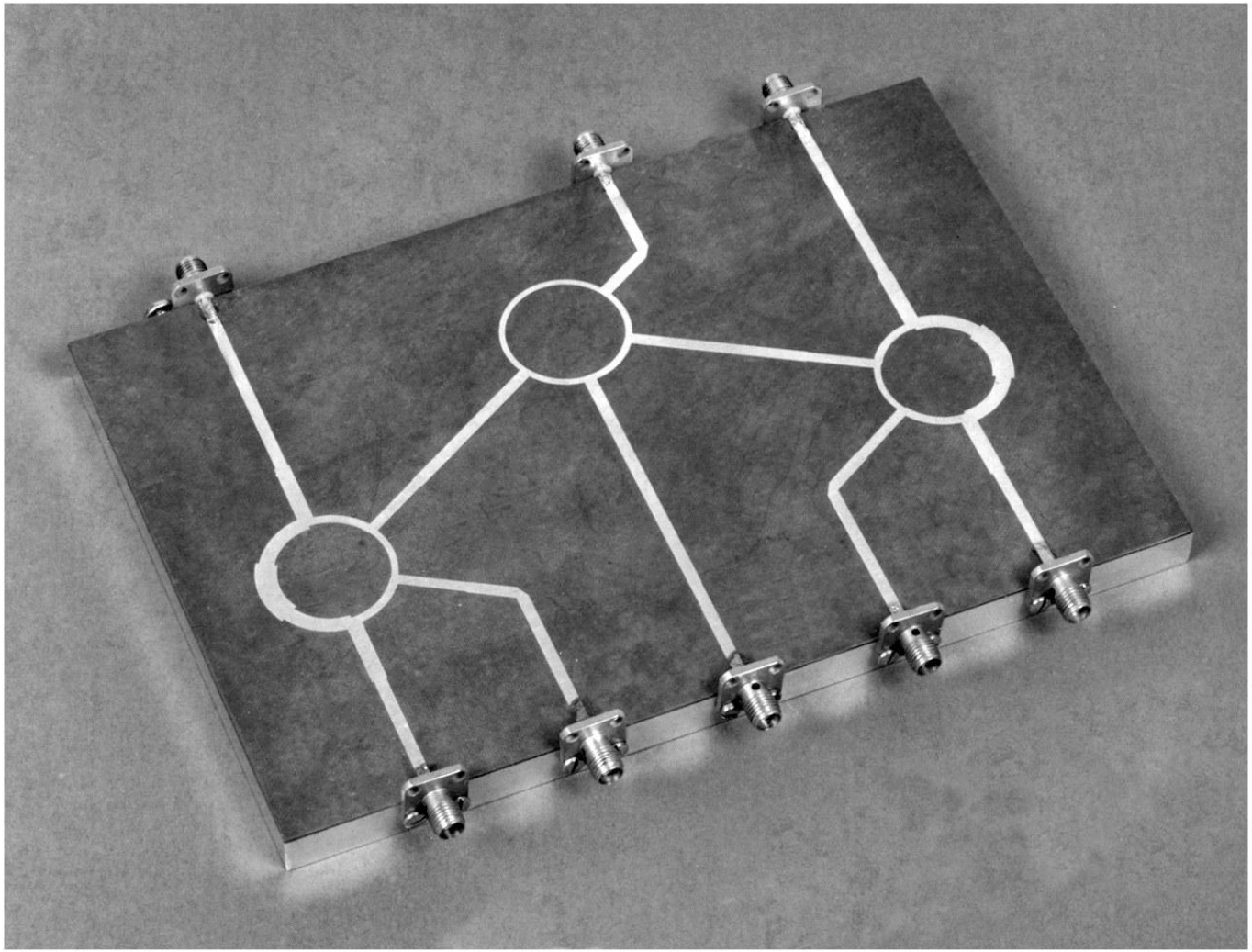
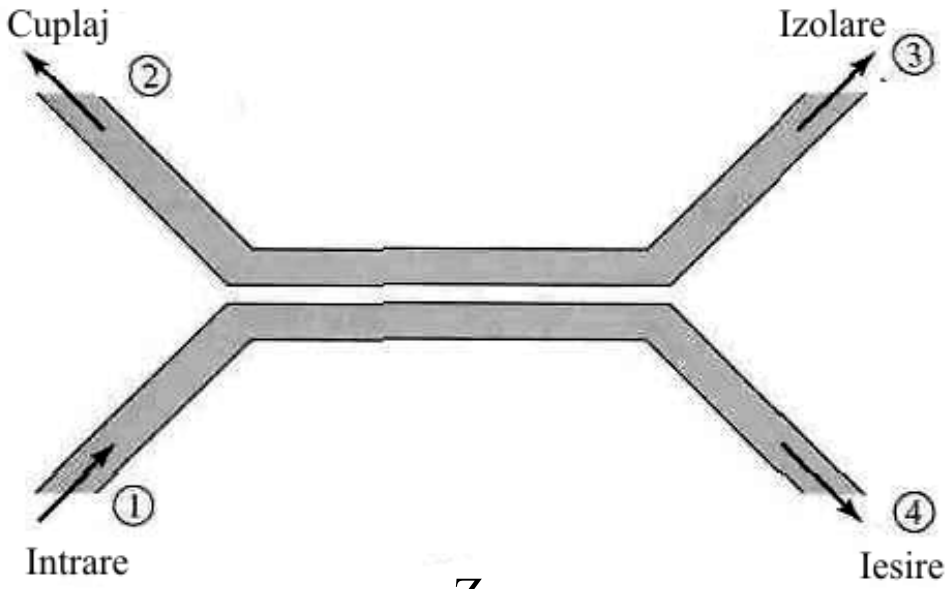
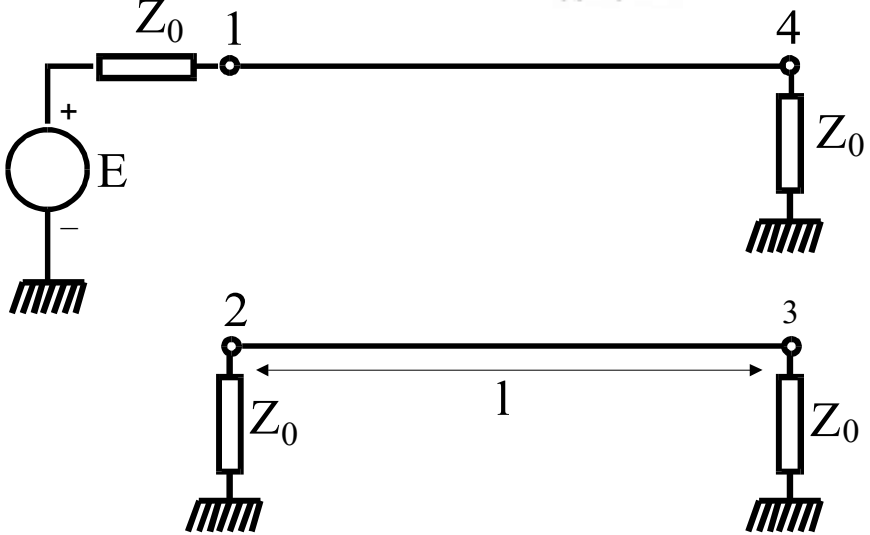


Figure 7.43
Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

Coupled Line Coupler





Coupled Lines

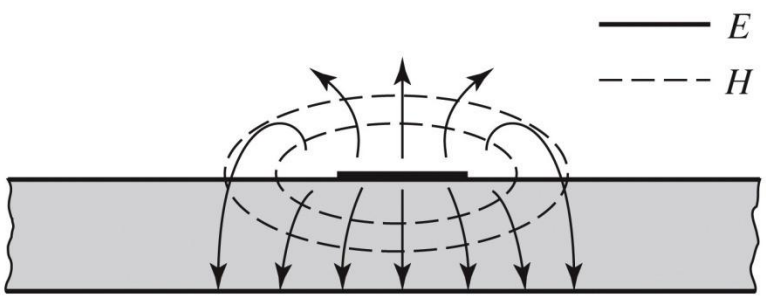
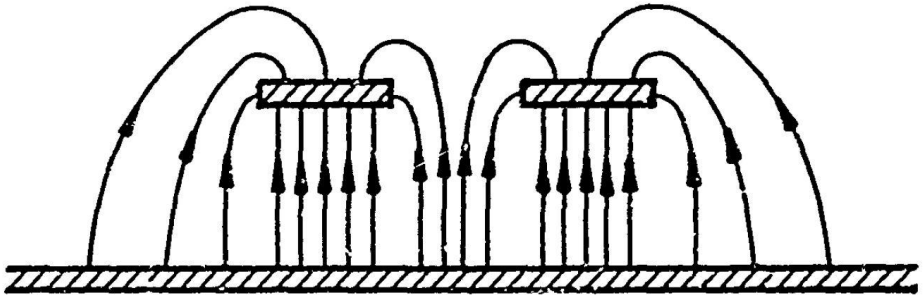
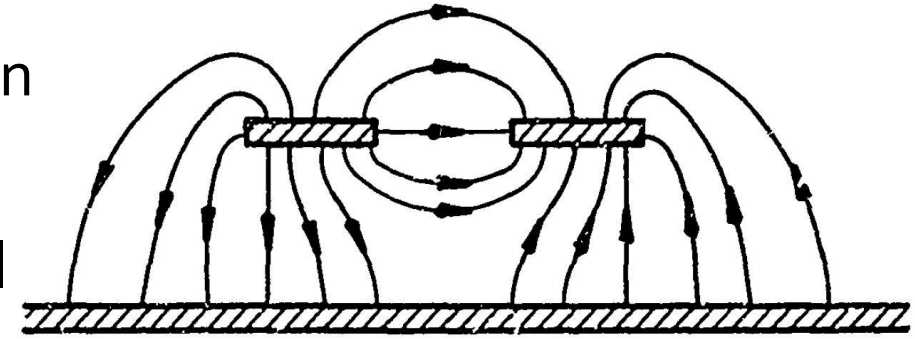


Figure 3.25b
© John Wiley & Sons, Inc. All rights reserved.



b) EVEN MODE ELECTRIC FIELD PATTERN (SCHEMATIC)

- Even mode - characterizes the common mode signal on the two lines
- Odd mode - characterizes the differential mode signal between the two lines
- Each of the two modes is characterized by **different** characteristic impedances



c) ODD MODE ELECTRIC FIELD PATTERN (SCHEMATIC)

Even/Odd Mode Analysis

- In linear circuits we can use the superposition principle
- advantages
  - reduction of the circuit complexity
  - decrease of the number of ports (**main** advantage)

Response (ODD + EVEN) = Response (ODD) + Response (EVEN)

We can benefit from existing symmetries !!

Coupled Lines

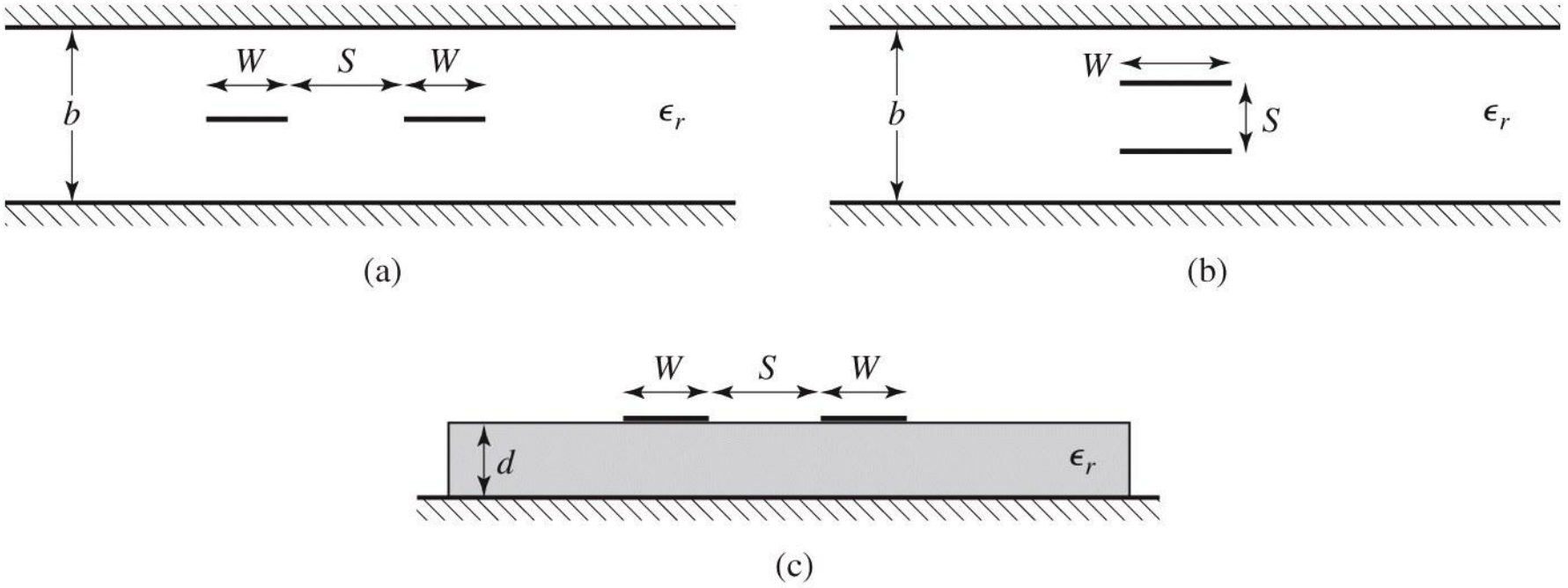


Figure 7.26

© John Wiley & Sons, Inc. All rights reserved.

Coupled Lines

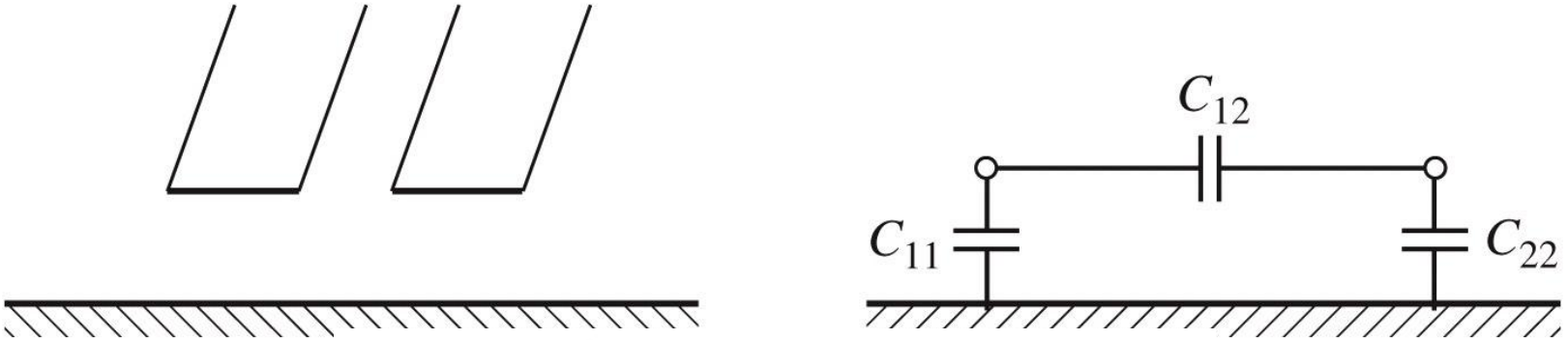
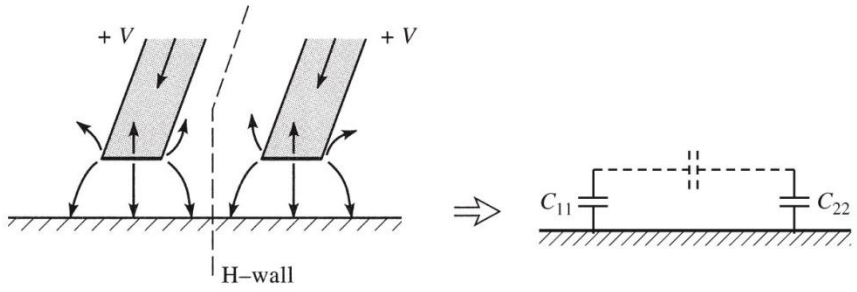
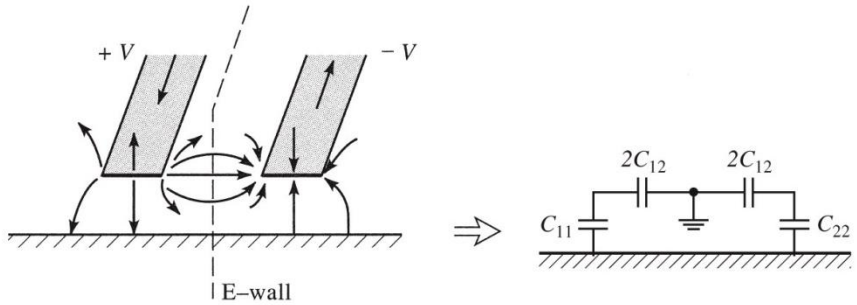


Figure 7.27
© John Wiley & Sons, Inc. All rights r



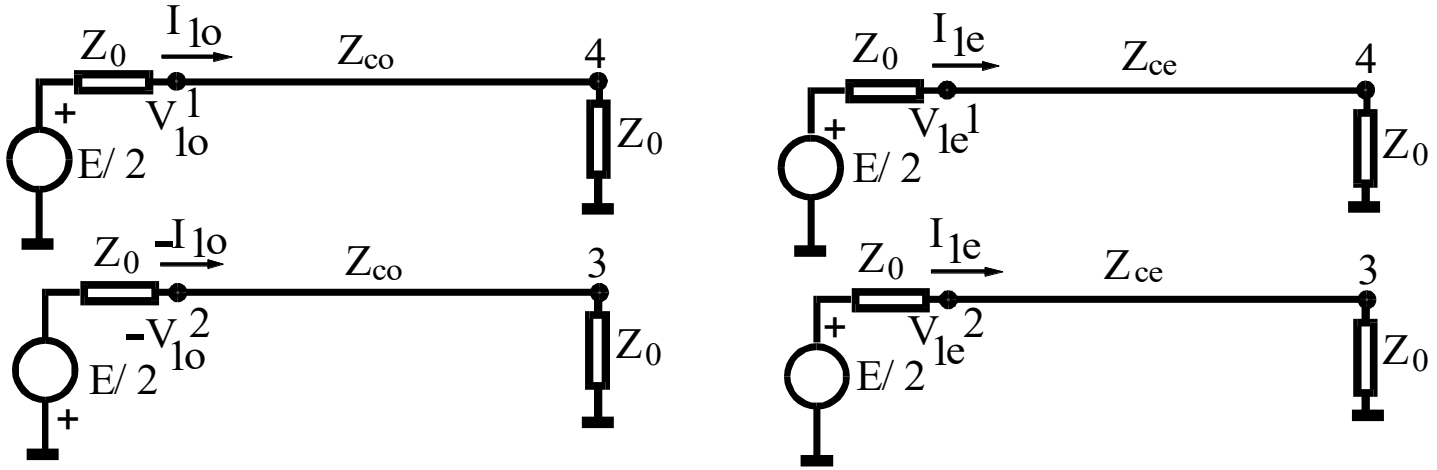
(a)



(b)

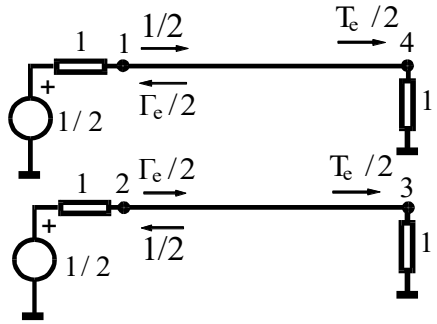
Figure 7.28
© John Wiley & Sons, Inc. All rights reserved.

Matching in Coupled Line Coupler

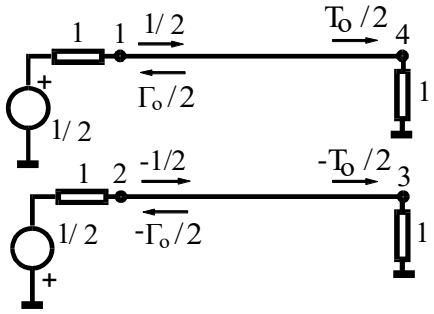


$$\begin{cases} Z_{ce}Z_{co} = Z_0^2 \\ \theta_e = \theta_o \end{cases}$$

Directivity and Coupling factor



modul par



modul impar

$$a_1 = a_{1e} + a_{1o} = 1\,,\; a_2 = a_3 = a_4 = 0$$

$$b_1 = \frac{1}{2}(\Gamma_e + \Gamma_o) = 0 \Leftrightarrow$$

$$b_2 = \frac{1}{2}(\Gamma_e - \Gamma_o) = \frac{jC\sin(\theta)}{\cos(\theta)\sqrt{1-C^2} + j\sin(\theta)}$$

$$b_3 = \frac{1}{2}(T_e - T_o) = 0$$

$$b_4 = \frac{1}{2}(T_e + T_o) = \frac{\sqrt{1-C^2}}{\cos(\theta)\sqrt{1-C^2} + j\sin(\theta)}$$

$$C = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}$$

$$\theta = \pi/2$$

$$[S] = \begin{bmatrix} 0 & C & 0 & -j\sqrt{1-C^2} \\ C & 0 & -j\sqrt{1-C^2} & 0 \\ 0 & -j\sqrt{1-C^2} & 0 & C \\ -j\sqrt{1-C^2} & 0 & C & 0 \end{bmatrix}$$

Coupled Line Coupler

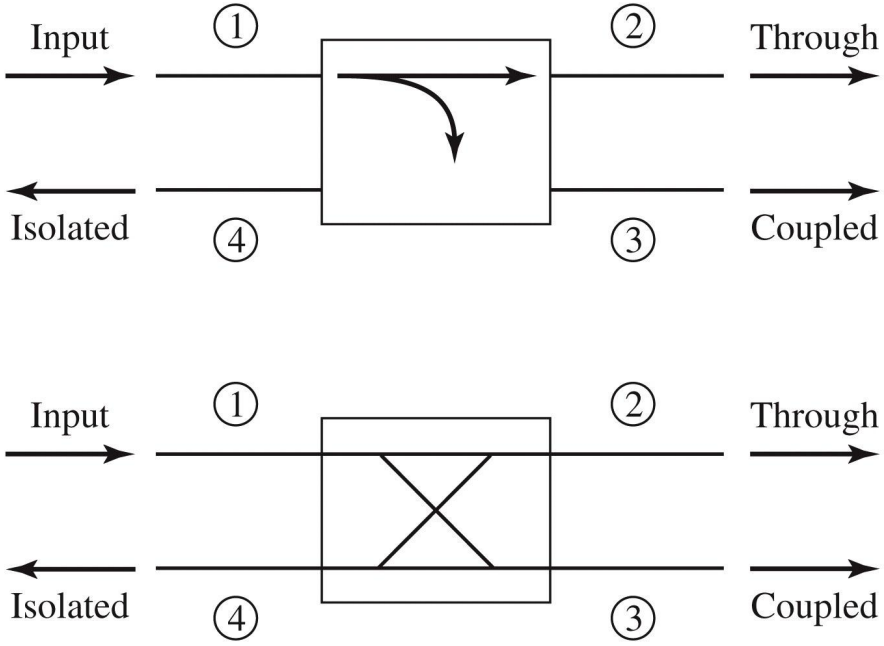
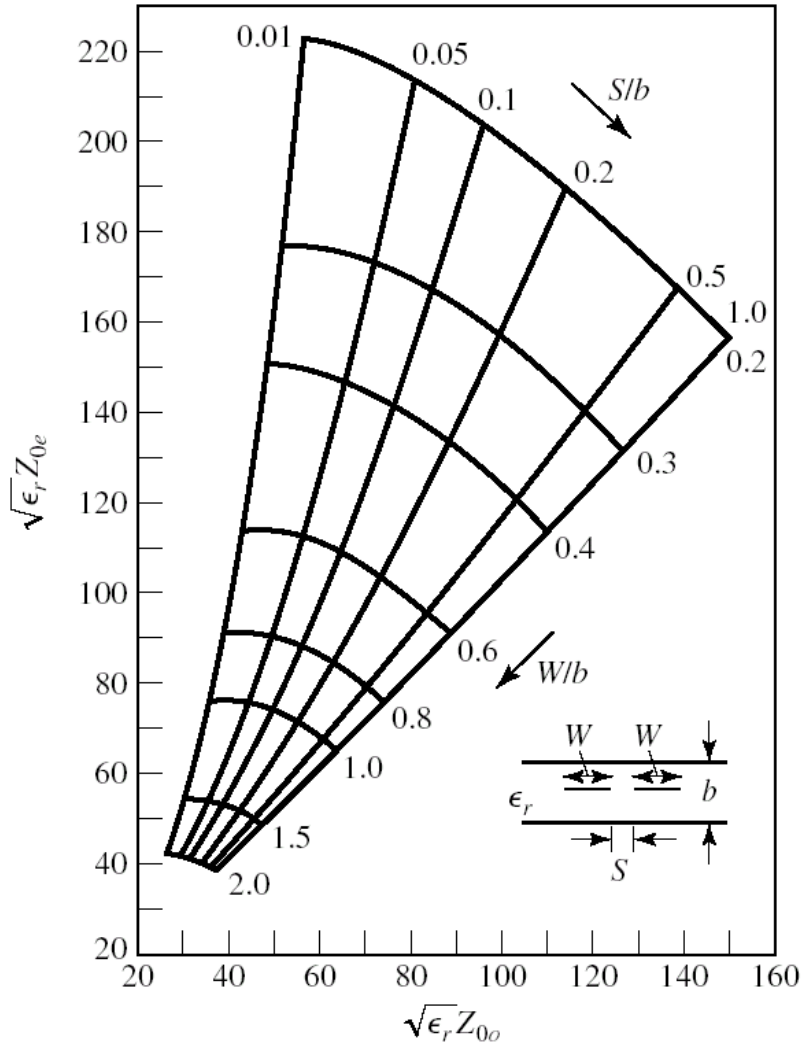


Figure 7.4
© John Wiley & Sons, Inc. All rights reserved.

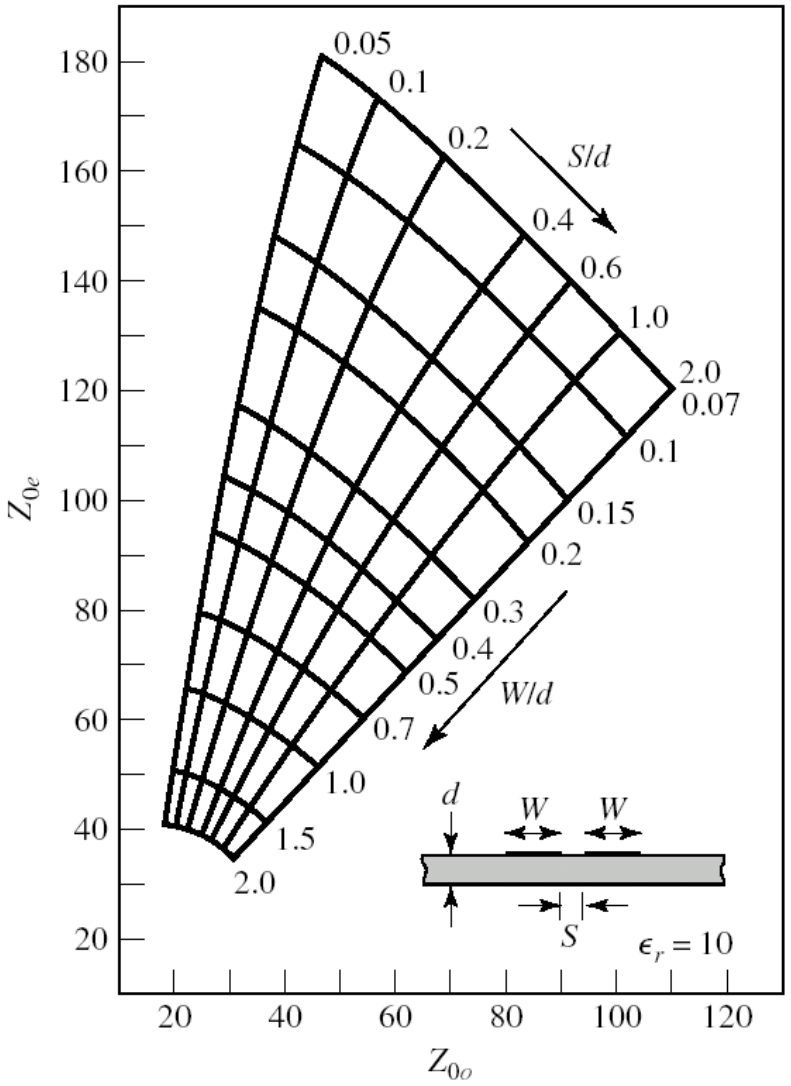
$$[S] = -j \cdot \begin{bmatrix} 0 & \sqrt{1-C^2} & jC & 0 \\ \sqrt{1-C^2} & 0 & 0 & jC \\ jC & 0 & 0 & \sqrt{1-C^2} \\ 0 & jC & \sqrt{1-C^2} & 0 \end{bmatrix}$$

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

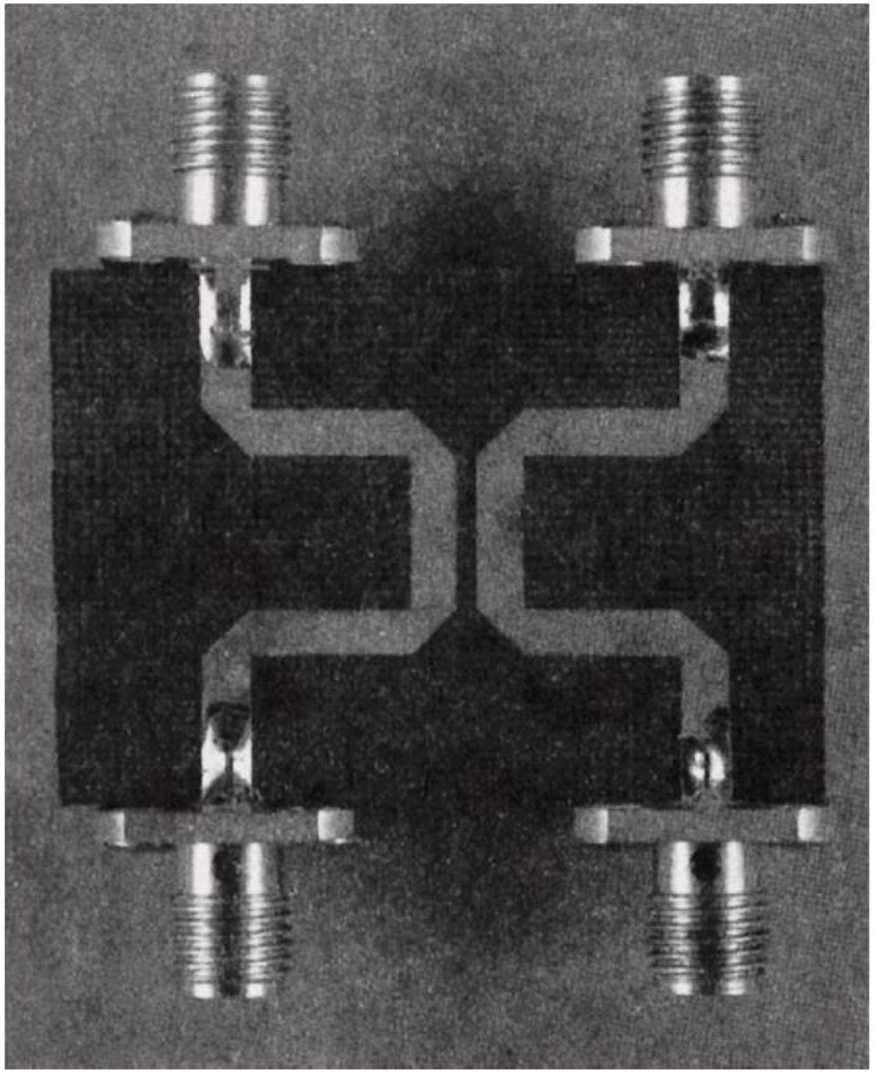
Normalized even- and odd-mode characteristic impedance design data for edge-coupled striplines.



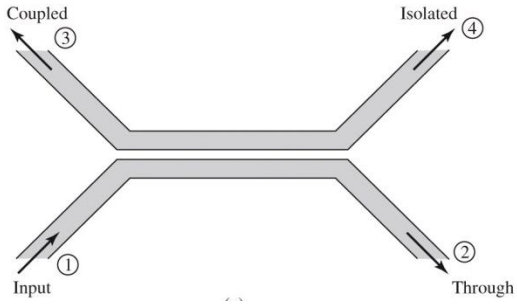
Even- and odd-mode characteristic impedance design data for coupled microstrip lines on a substrate with $\varepsilon_r = 10$.



Coupled Line Coupler



Coupled Line Coupler



$$Z_{ce}Z_{co} = Z_0^2$$

$$|\beta| = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}$$

$$C\,[\text{dB}] = -20 \cdot \log_{10}\left(\frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}\right)$$

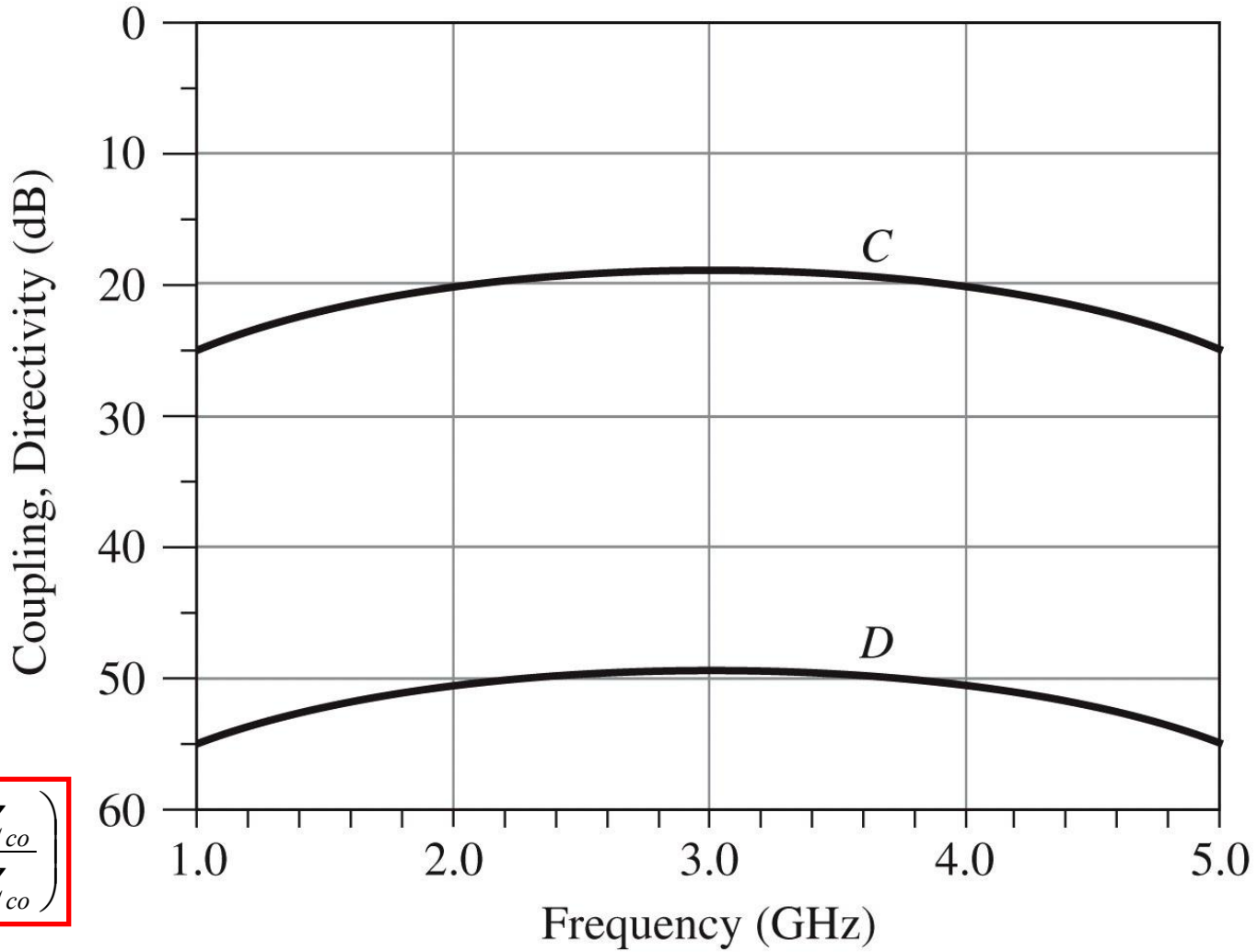


Figure 7.34

© John Wiley & Sons, Inc. All rights reserved.

Example

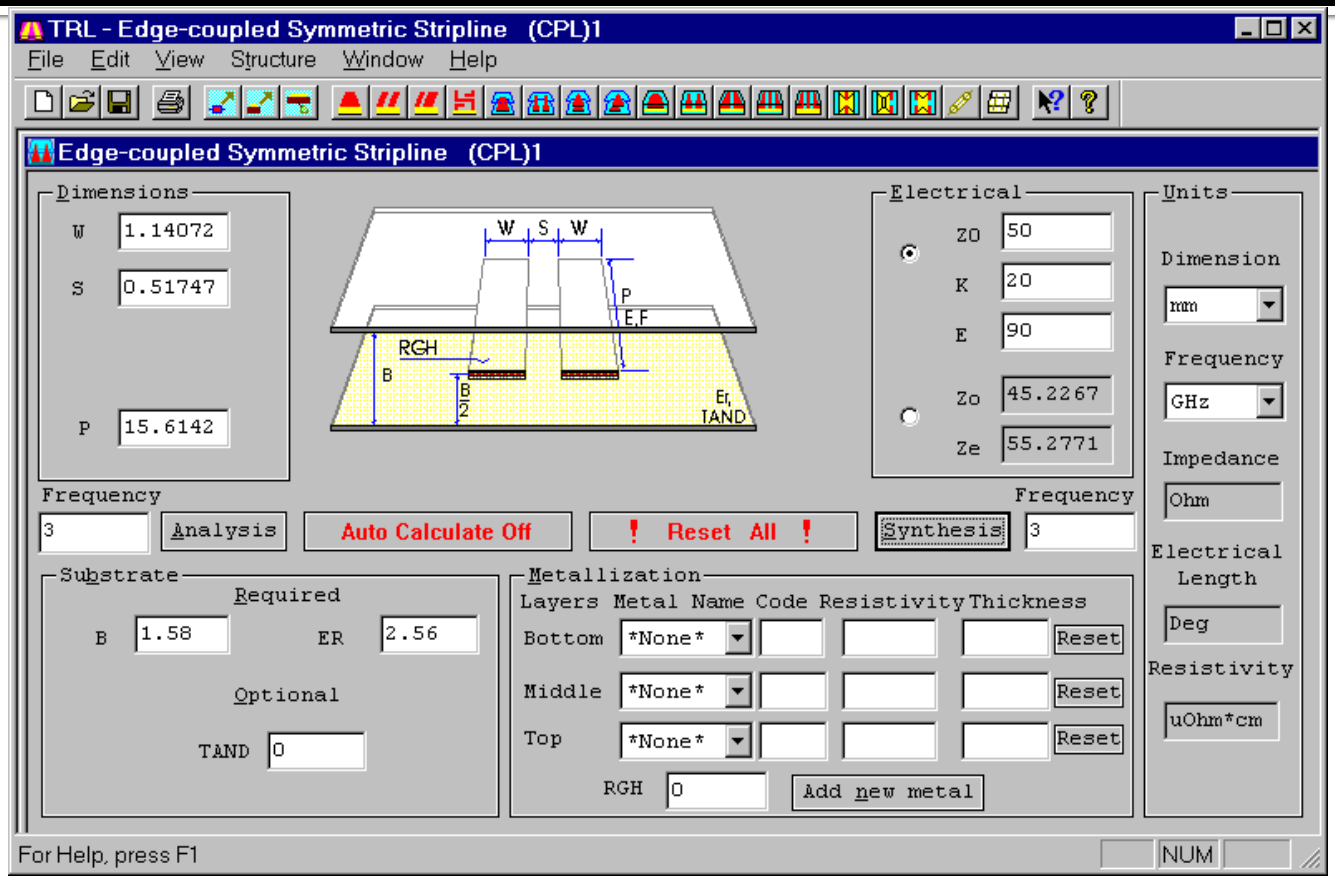
Design a coupled line coupler with 20 dB coupling factor, using stripline technology, with a distance between ground planes of 0.158 cm and an electrical permittivity of 2.56, working on 50Ω, at the design frequency of 3 GHz. Plot the coupling and directivity between 1 and 5 GHz.

Solution

$$C = 10^{-20/20} = 0.1$$

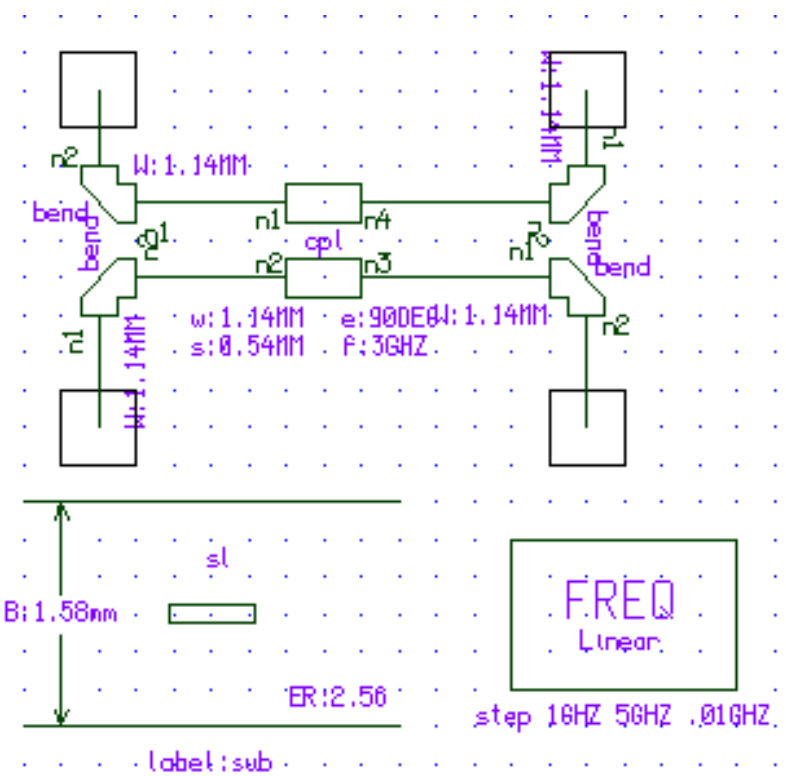
$$Z_{co} = 50\sqrt{\frac{0.9}{1.1}} = 45.23\Omega$$

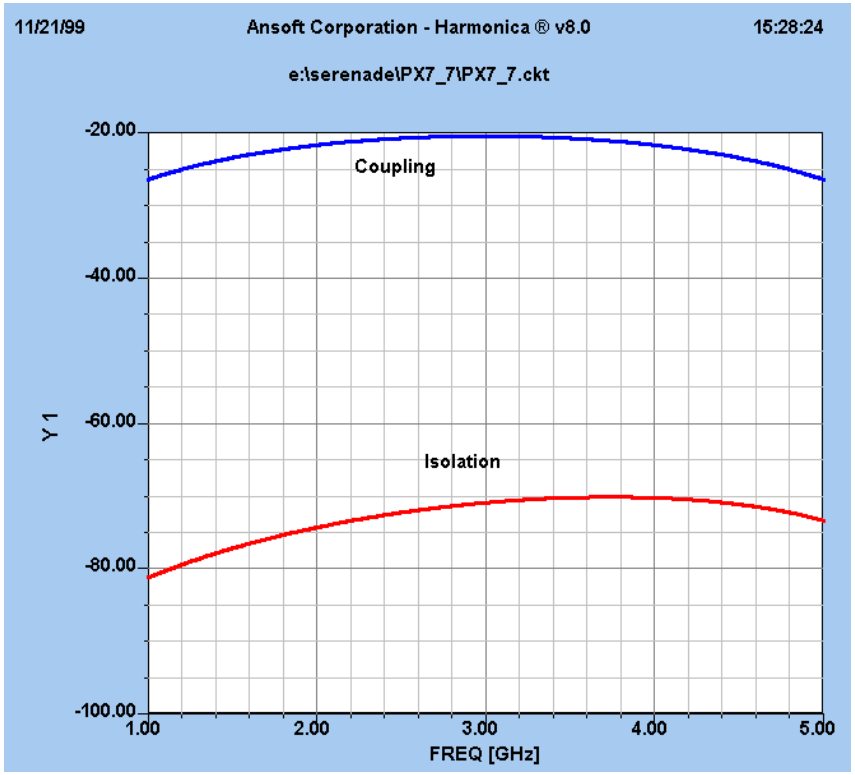
$$Z_{ce} = 50\sqrt{\frac{1.1}{0.9}} = 55.28\Omega$$



$$Z_{ce} = Z_0\sqrt{\frac{1+C}{1-C}}\,,\; Z_{co} = Z_0\sqrt{\frac{1-C}{1+C}}$$

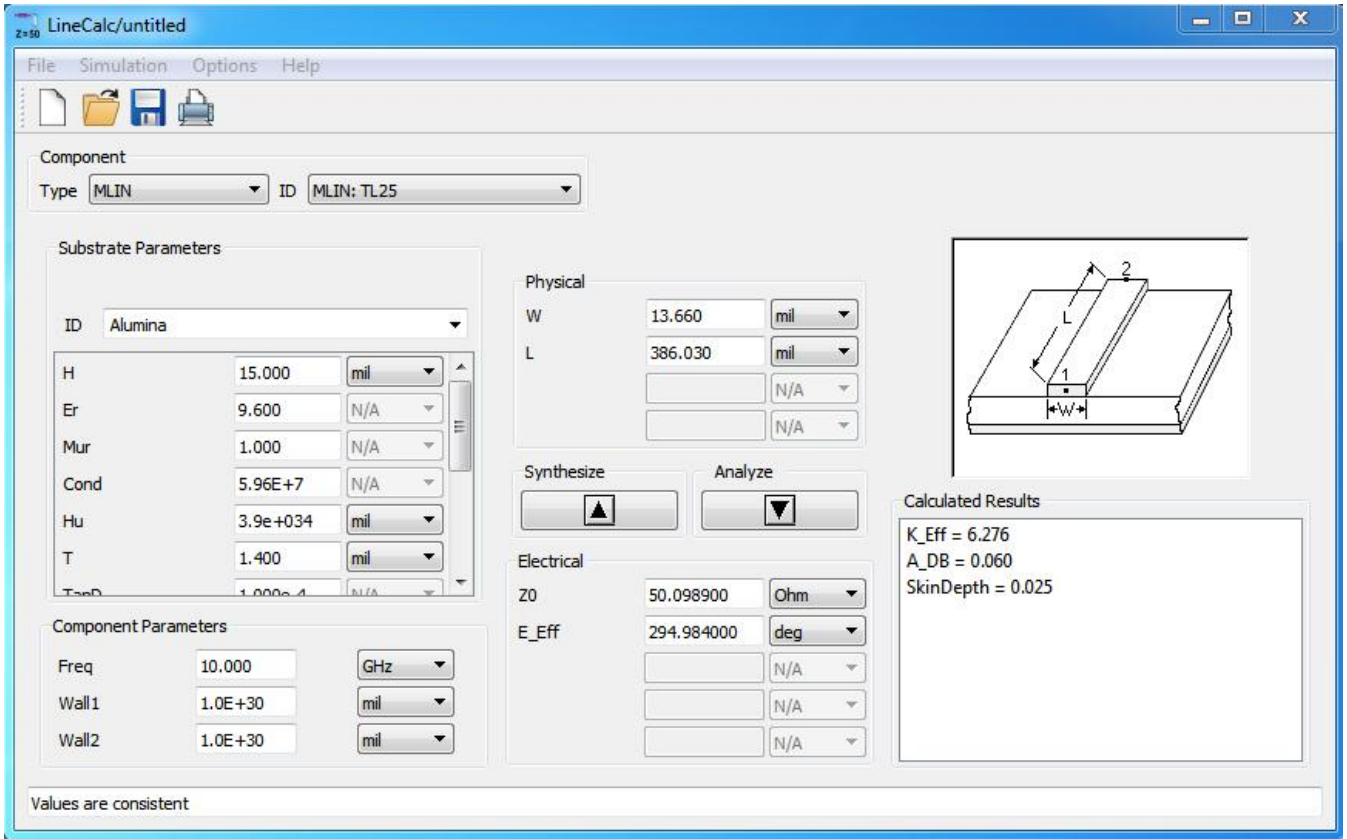
Simulation


W:1.14MM
bend
cpl
n1
n2
n3
n4
w:1.14MM
s:0.54MM
e:90DEG
F:3GHZ
W:1.14MM
sl
B:1.58mm
ER:2.56
FREQ
Linear
step 1GHZ 5GHZ .01GHZ
label:sub


11/21/99
Ansoft Corporation - Harmonica ® v8.0
15:28:24
e:\serenade\PX7_7\PX7_7.ckt
Coupling
Isolation
-20.00
-40.00
-60.00
-80.00
-100.00
Y1
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]

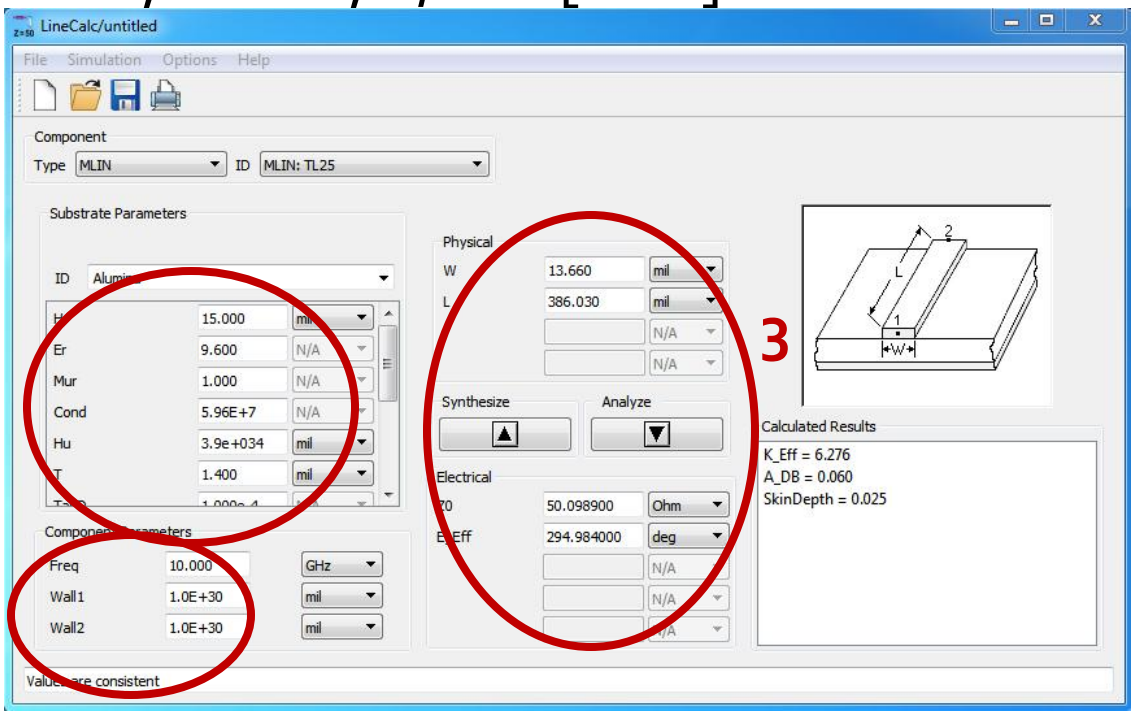
ADS linecalc

- In schematics: >Tools>LineCalc>Start
- for Microstrip lines >Tools>LineCalc>Send to Linecalc



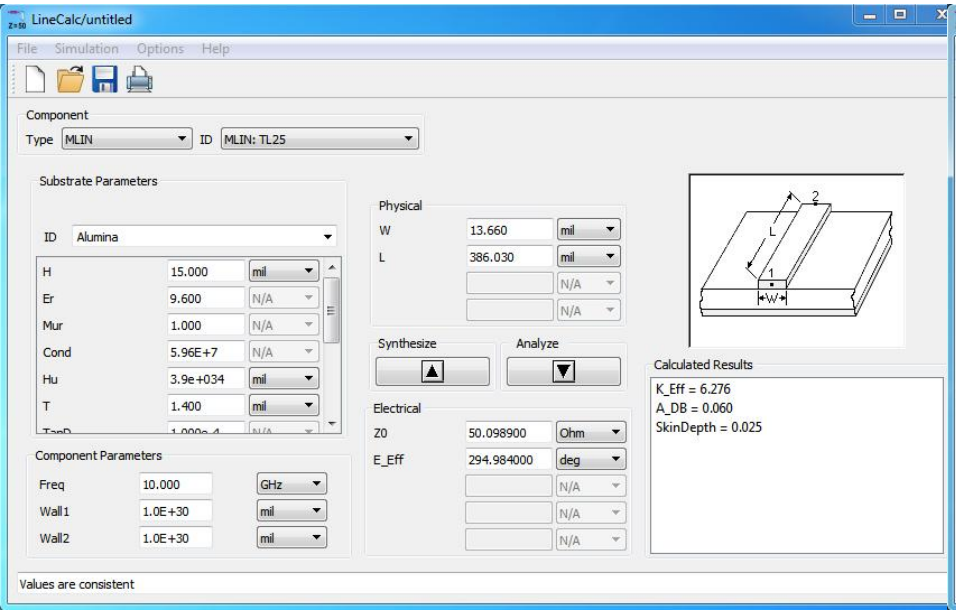
ADS linecalc

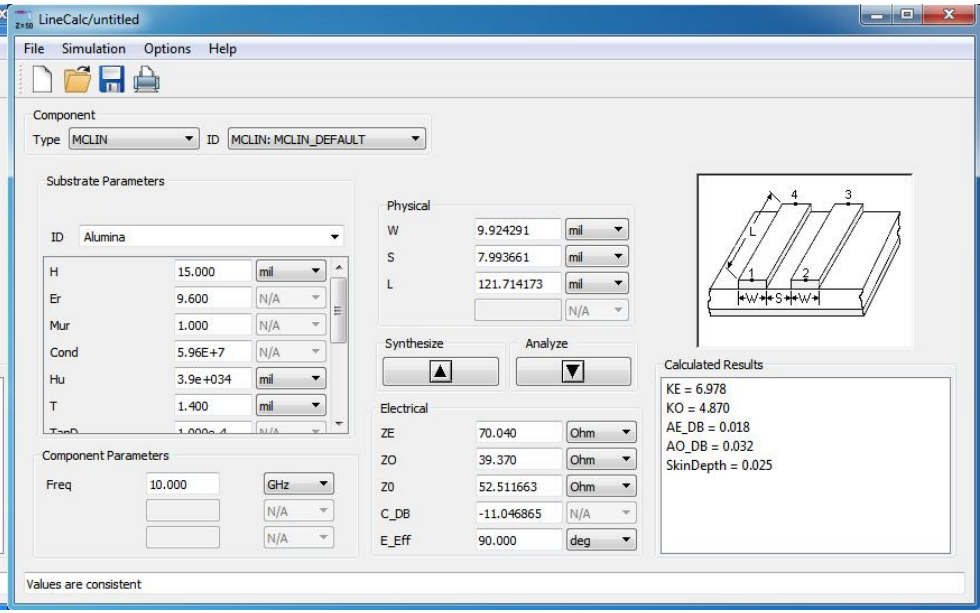
- 1. Define substrate (receive from schematic)
- 2. Insert frequency
- 3. Insert input data
  - Analyze: W,L → Zo,E or Ze,Zo,E / at f [GHz]
  - Synthesis: Zo,E → W,L / at f [GHz]



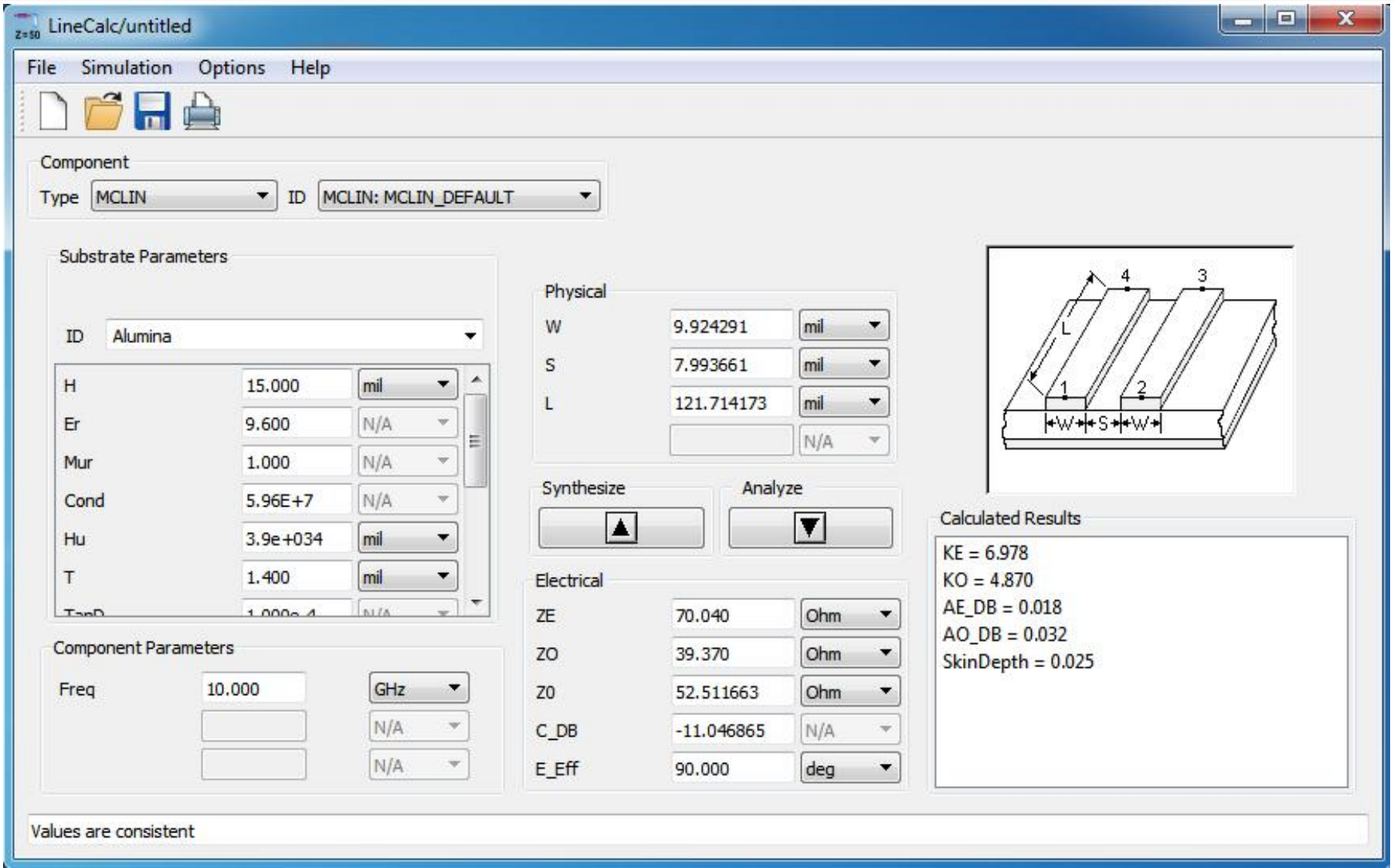
ADS linecalc

- Can be used for:
  - microstrip lines MLIN: W,L ⇔ Zo,E
  - microstrip coupled lines MCLIN: W,L,S ⇔ Ze,Zo,E





ADS linecalc



Multisection Coupled Line Couplers

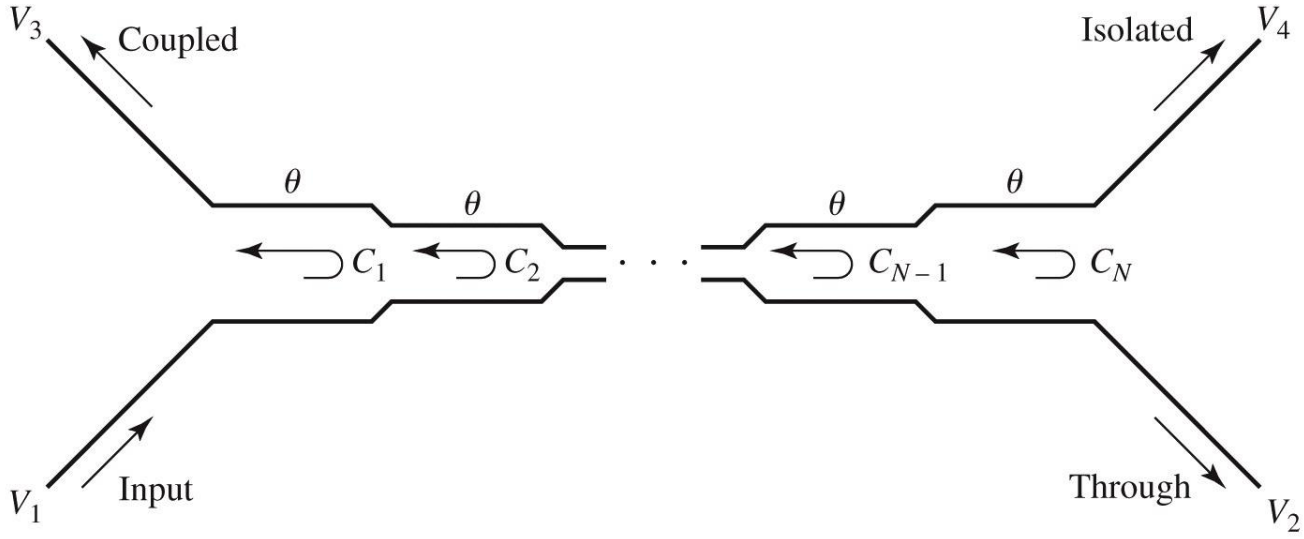


Figure 7.35
© John Wiley & Sons, Inc. All rights reserved.

C<<1

$$\frac{V_3}{V_1} = b_3 = \frac{jC\sin\theta}{\cos\theta\sqrt{1-C^2} + j\sin\theta} = \frac{jCtg\theta}{\sqrt{1-C^2} + jtg\theta} \approx \frac{jCtg\theta}{1+jtg\theta} = jC\sin\theta e^{-j\theta}$$

$$\frac{V_2}{V_1} = b_2 = \frac{\sqrt{1-C^2}}{\cos\theta\sqrt{1-C^2} + j\sin\theta} \approx \frac{1}{\cos\theta + j\sin} = e^{-j\theta}$$

$$C = \frac{V_3}{V_1} = 2j\sin\theta e^{-j\theta} e^{-j(N-1)\theta}\left[C_1\cos(N-1)\theta + C_2\cos(N-3)\theta + \ldots + \frac{1}{2}C_{\frac{N+1}{2}}\right]$$

Example

Design a three sections coupled line coupler with 20 dB coupling factor, binomial characteristic (maximum flat), working on 50Ω, at the design frequency of 3 GHz. Plot the coupling and directivity between 1 and 5 GHz

Solution

$$\left.\frac{d^n}{d\theta^n}C(\theta)\right|_{\theta=\pi/2}=0,\ n=1,2$$

$$C=\left|\frac{V_3}{V_1}\right|=2\sin\theta\left[C_1\cos 2\theta+\frac{1}{2}C_2\right]=C_1(\sin 3\theta-\sin\theta)+C_2\sin\theta$$

$$\frac{dC}{d\theta}=\left[3C_1\cos 3\theta+(C_2-C_1)\cos\theta\right]|_{\theta=\pi/2}=0$$

$$\frac{d^2C}{d\theta^2}=\left[-9C_1\sin 3\theta-(C_2-C_1)\sin\theta\right]|_{\theta=\pi/2}=10C_1-C_2=0$$

$$\begin{cases}C_2-2C_1=0.1\\10C_1-C_2=0\end{cases}$$

$$\begin{cases}C_1=C_3=0.0125\\C_2=0.125\end{cases}$$

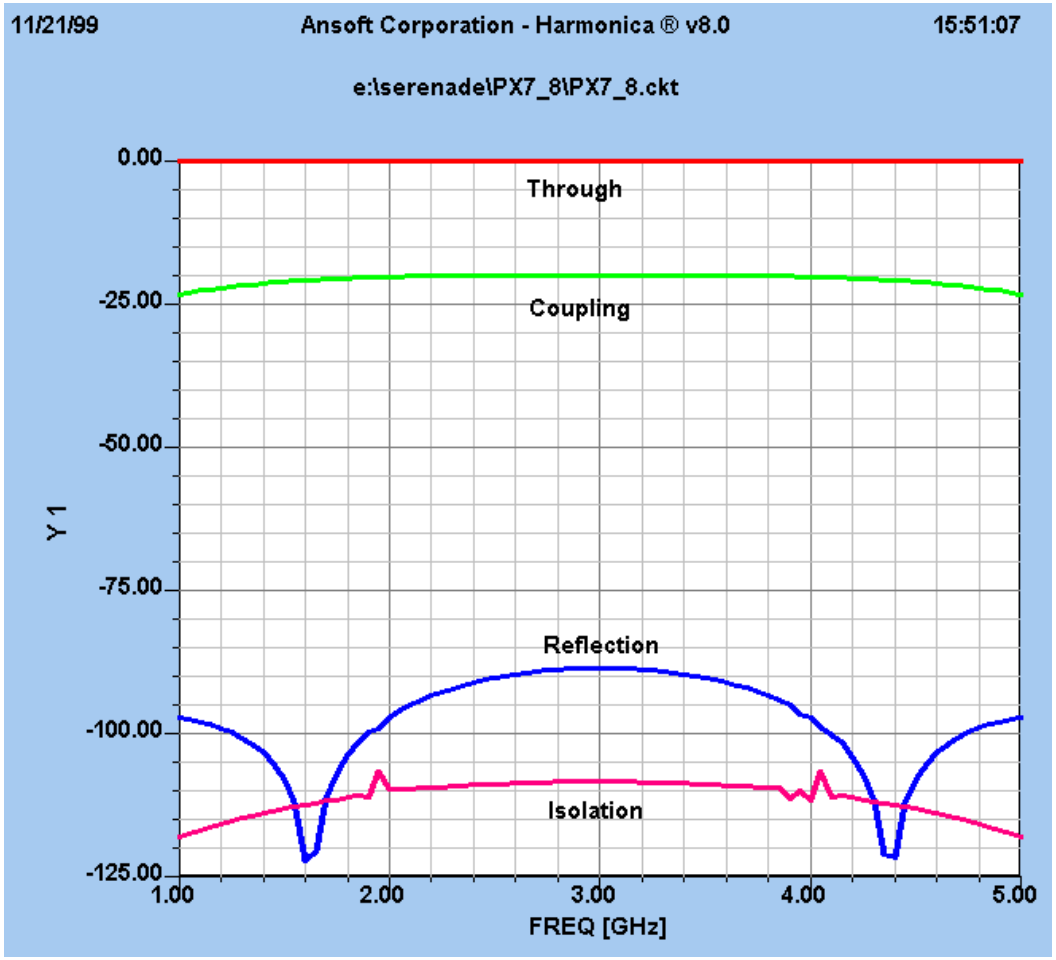
$$Z_{0e}^1=Z_{0e}^3=50\sqrt{\frac{1.0125}{0.9875}}=50.63\,\Omega$$

$$Z_{0o}^1=Z_{0o}^3=50\sqrt{\frac{0.9875}{1.0125}}=49.38\,\Omega$$

$$Z_{0e}^2=50\sqrt{\frac{1.125}{0.875}}=56.69\,\Omega$$

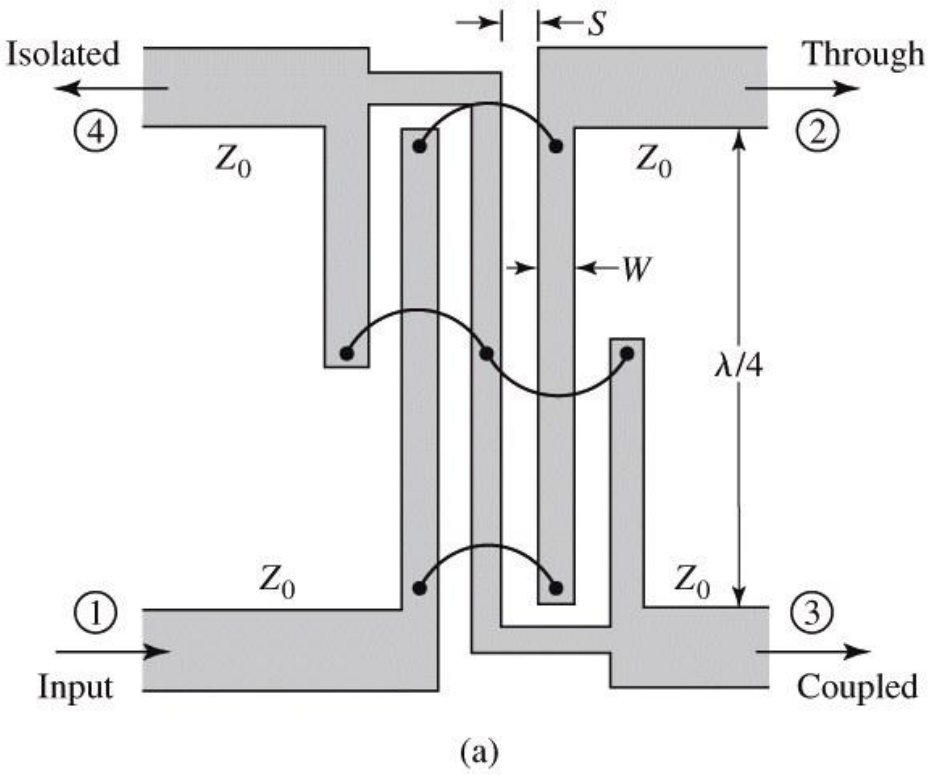
$$Z_{0o}^2=50\sqrt{\frac{0.875}{1.125}}=44.10\,\Omega$$

Simulare


11/21/99
Ansoft Corporation - Harmonica ® v8.0
15:51:07
e:\serenade\PX7_8\PX7_8.ckt
0.00
-25.00
-50.00
-75.00
-100.00
-125.00
Y 1
Through
Coupling
Reflection
Isolation
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]

The Lange Coupler

- allows achieving coupling factors of 3 or 6 dB



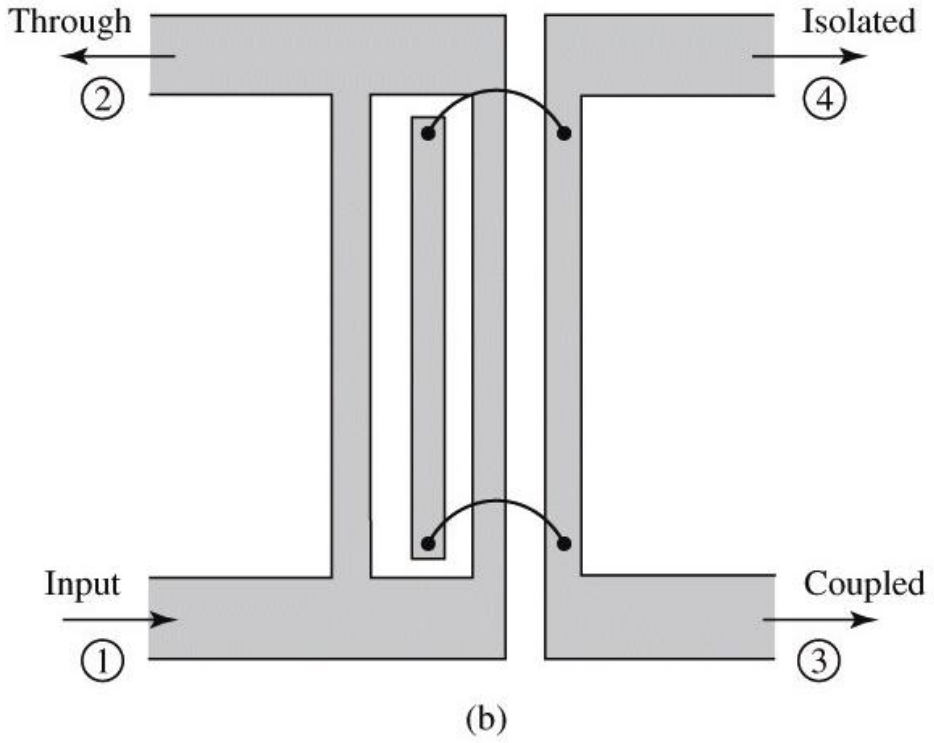


Figure 7.38

© John Wiley & Sons, Inc. All rights reserved.

The Lange Coupler

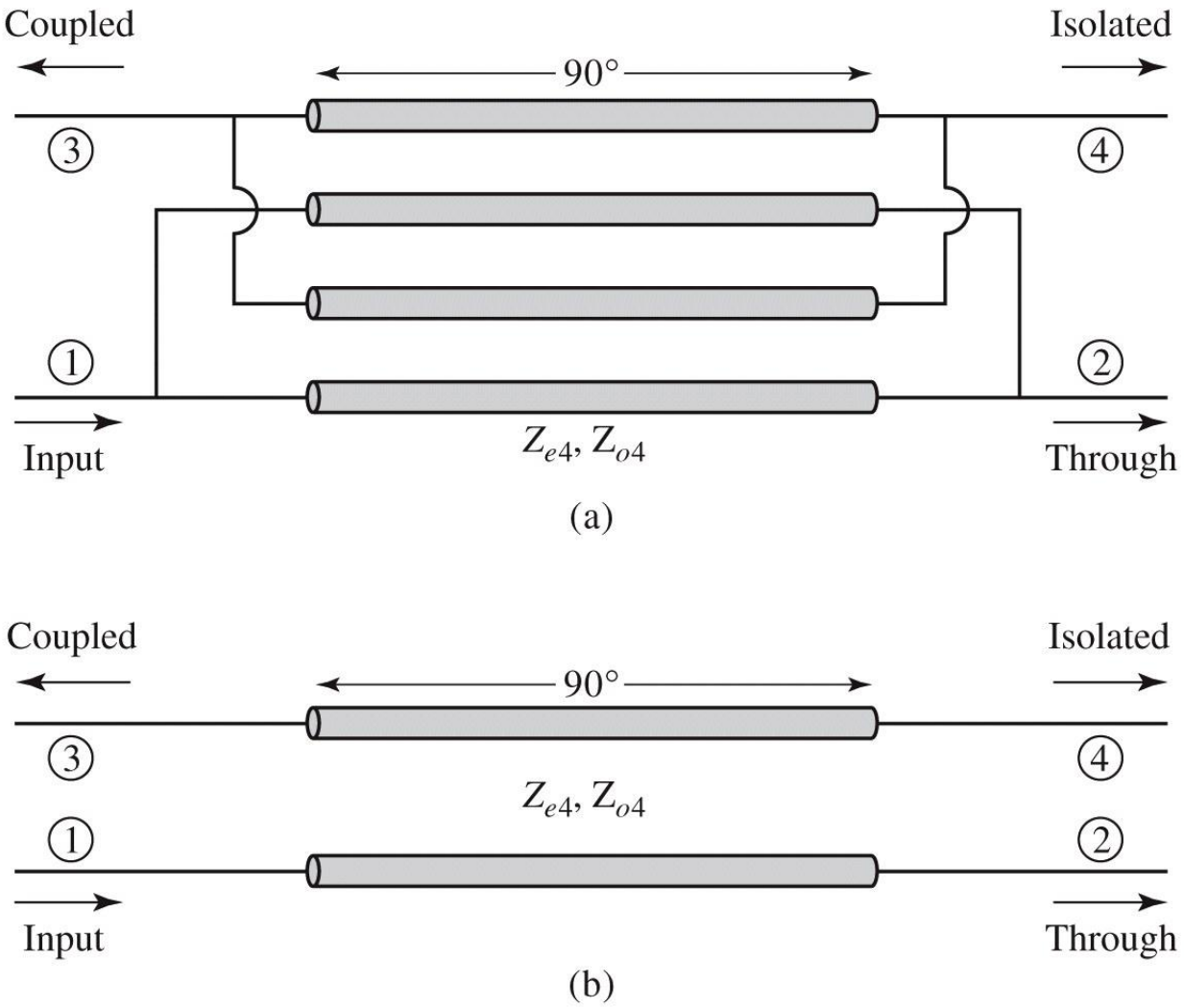
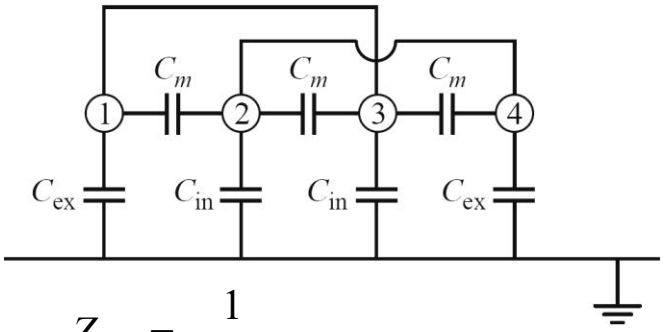


Figure 7.39

© John Wiley & Sons, Inc. All rights reserved.

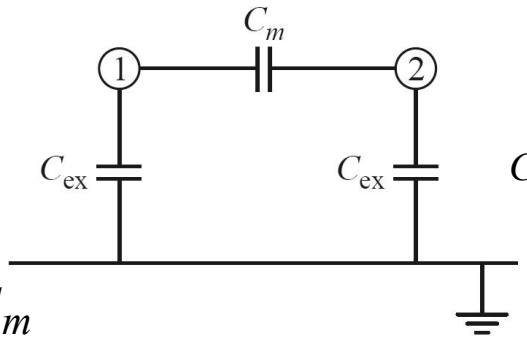
Circuit model



$$C_{in} = C_{ex} - \frac{C_{ex}C_m}{C_{ex} + C_m}$$

$$C_{e4} = C_{ex} + C_{in}$$

$$C_{o4} = C_{ex} + C_{in} + 6C_m$$



$$C_e = C_{ex}$$

$$C_o = C_{ex} + 2C_m$$

$$Z_{e4} = \frac{1}{vC_{e4}}$$

$$Z_{o4} = \frac{1}{vC_{o4}}$$

$$Z_0 = \sqrt{Z_{e4}Z_{o4}} = \sqrt{\frac{Z_{0e}Z_{0o}(Z_{0o} + Z_{0e})^2}{(3Z_{0o} + Z_{0e})(3Z_{0e} + Z_{0o})}}$$

$$C_{e4} = \frac{C_e(3C_e + C_o)}{C_e + C_o}$$

$$C_{o4} = \frac{C_o(3C_o + C_e)}{C_e + C_o}$$

$$Z_{e4} = Z_{0e}\frac{Z_{0e} + Z_{0o}}{3Z_{0o} + Z_{0e}}$$

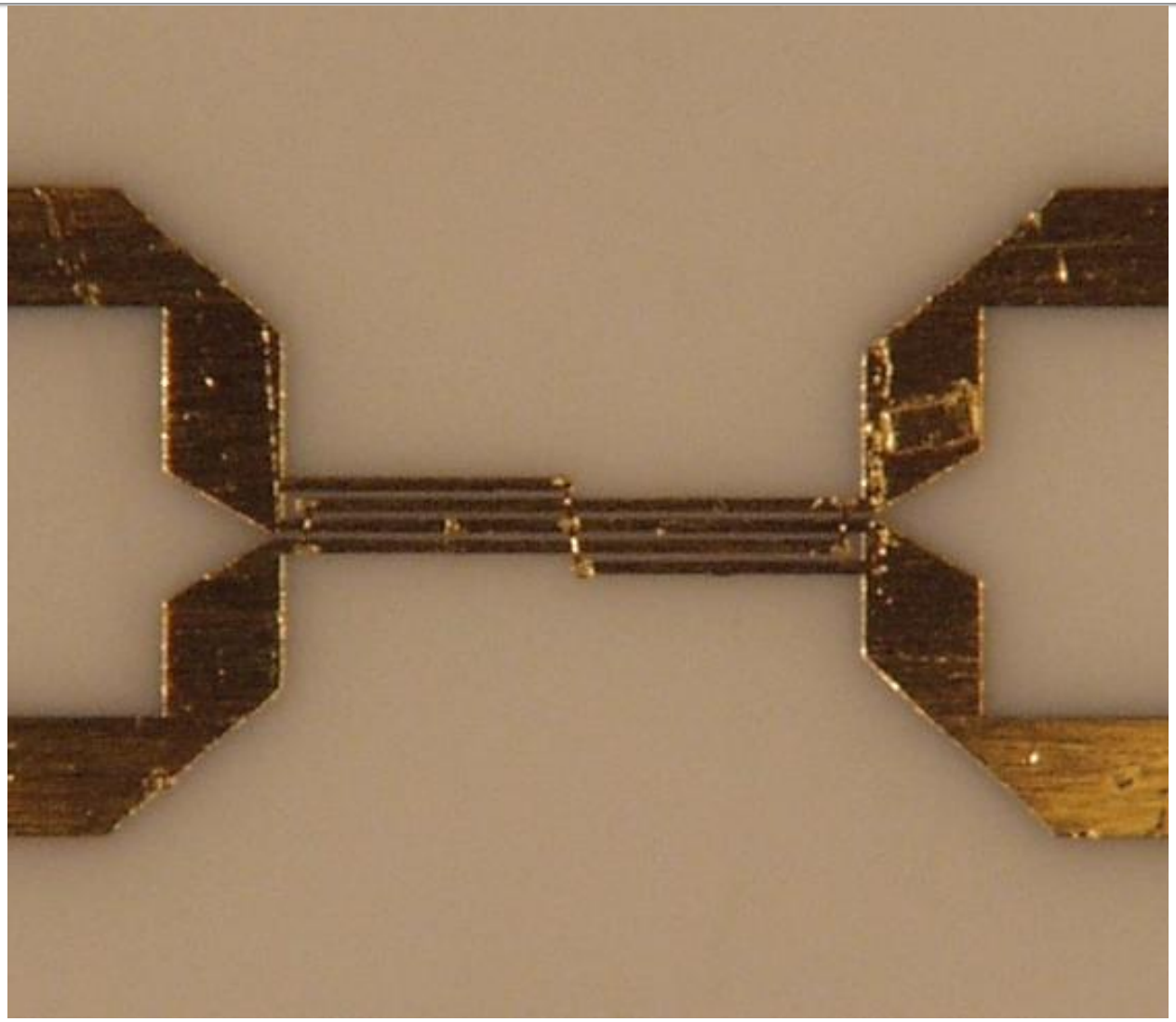
$$Z_{o4} = Z_{0o}\frac{Z_{0e} + Z_{0o}}{3Z_{0e} + Z_{0o}}$$

$$C = \frac{Z_{e4} - Z_{o4}}{Z_{e4} + Z_{o4}} = \frac{3\left(Z_{0e}^2 - Z_{0o}^2\right)}{3\left(Z_{0e}^2 + Z_{0o}^2\right) + 2Z_{0e}Z_{0o}}$$

$$Z_{0e} = \frac{4C - 3 + \sqrt{9 - 8C^2}}{2C\sqrt{(1-C)/(1+C)}}Z_0$$

$$Z_{0o} = \frac{4C + 3 - \sqrt{9 - 8C^2}}{2C\sqrt{(1+C)/(1-C)}}Z_0$$

The Lange Coupler



Directional Couplers

Laboratory no. 2

Directional Coupler

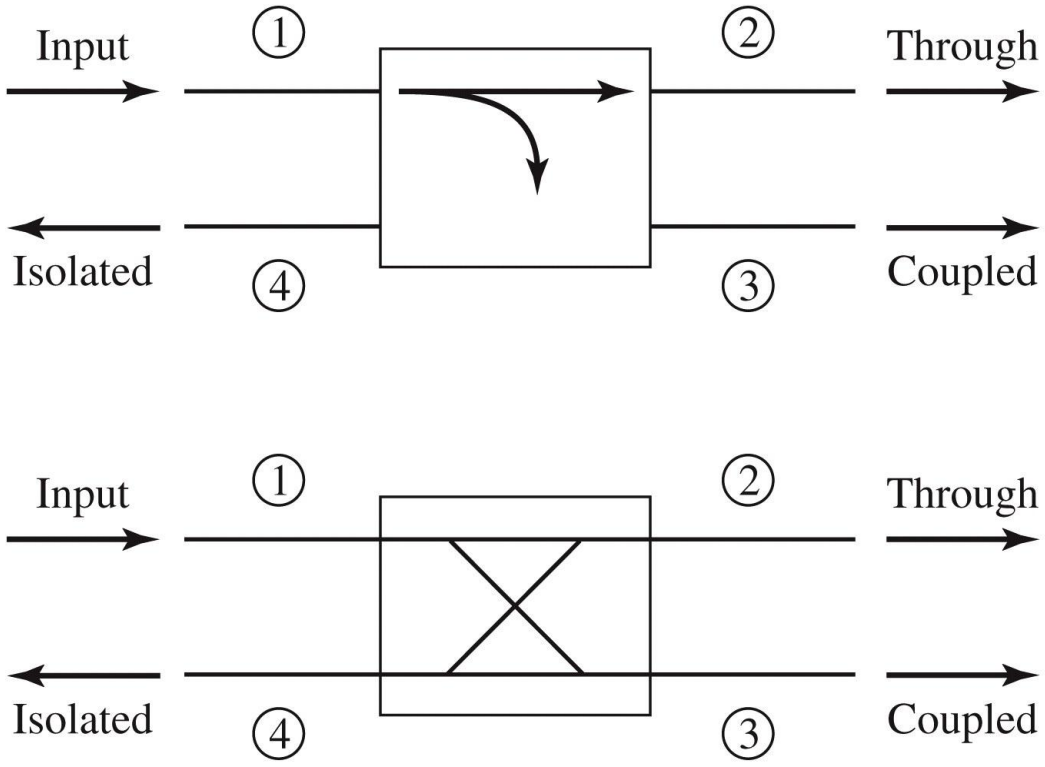


Figure 7.4
© John Wiley & Sons, Inc. All rights reserved.

$$|S_{12}|^2 = \alpha^2 = 1 - \beta^2$$

$$|S_{13}|^2 = \beta^2$$

Cuplaj

$$C = 10\log\frac{P_1}{P_3} = -20\cdot\log(\beta)\,[\text{dB}]$$

Directivitate

$$D = 10\log\frac{P_3}{P_4} = 20\cdot\log\left(\frac{\beta}{|S_{14}|}\right)[\text{dB}]$$

Izolare

$$I = 10\log\frac{P_1}{P_4} = -20\cdot\log|S_{14}|\,[\text{dB}]$$

$$I = D + C\ ,\ \ dB$$

The cuadrature (90°) hybrid

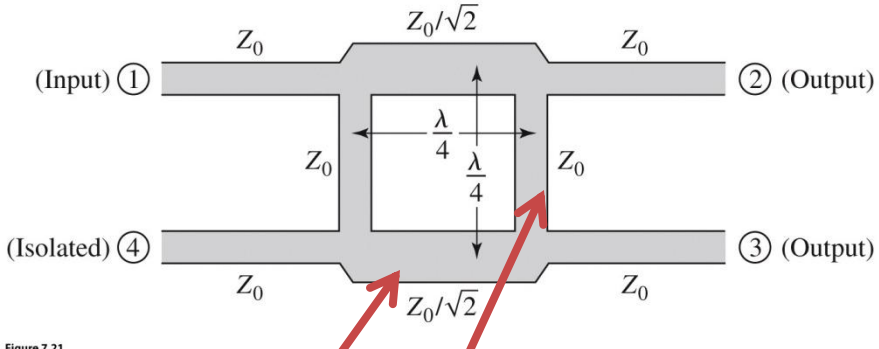


Figure 7.21
© John Wiley & Sons, Inc. All rights reserved.

$$y_2^2 = 1 + y_1^2$$

$$|\beta| = \frac{\sqrt{y_2^2 - 1}}{y_2}$$

$$C[\text{dB}] = -20 \cdot \log_{10} \frac{\sqrt{y_2^2 - 1}}{y_2}$$

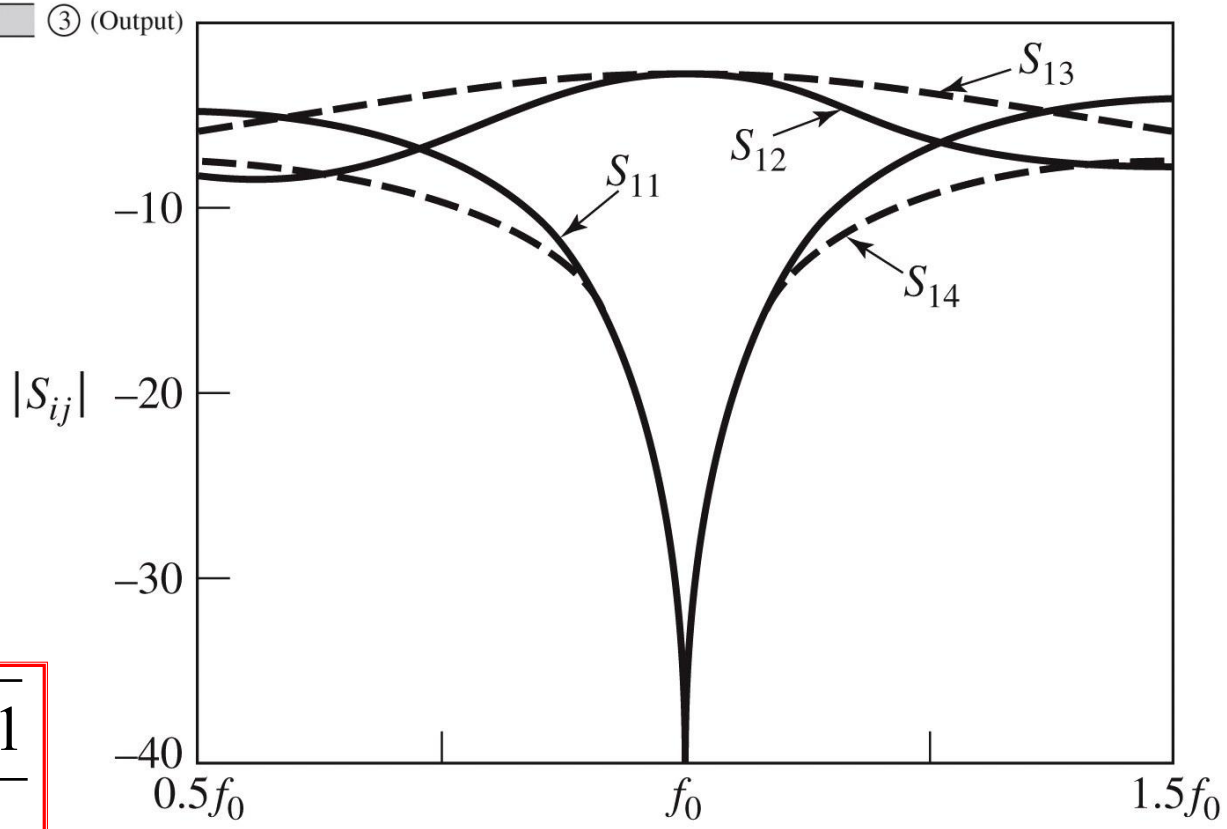
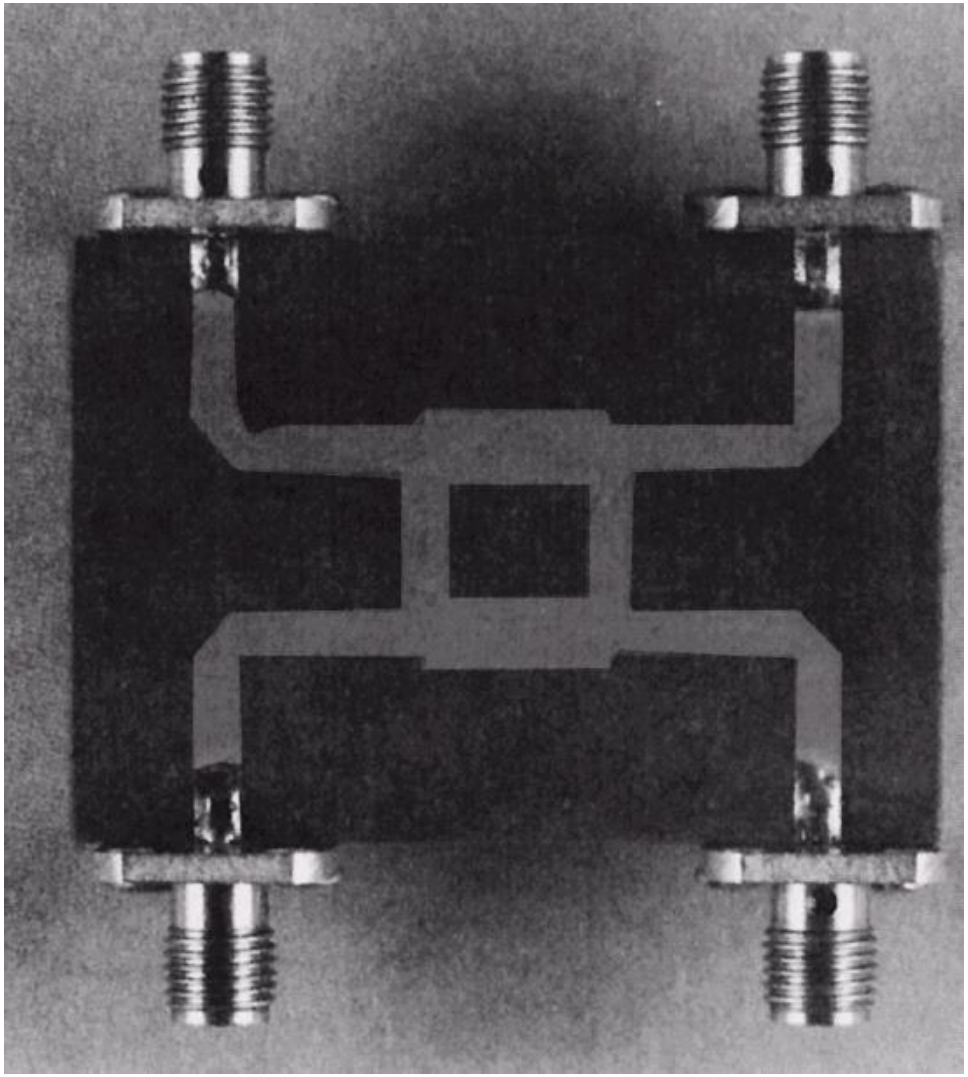
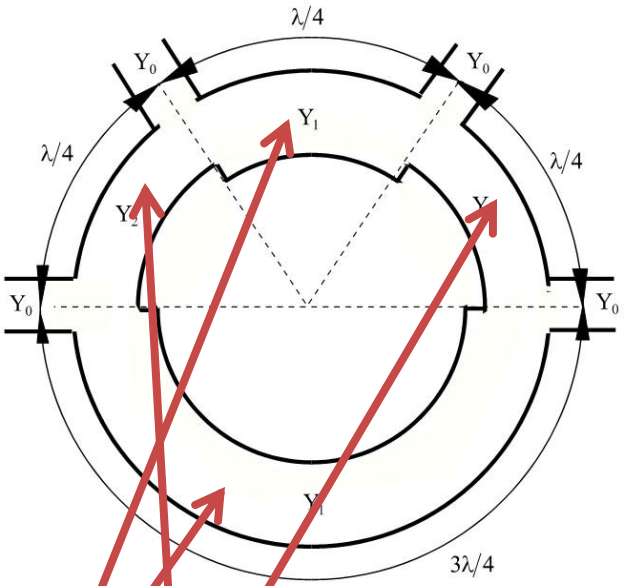


Figure 7.25
© John Wiley & Sons, Inc. All rights reserved.

Quadrature coupler



The 180° ring hybrid (rat-race)



$$y_1^2 + y_2^2 = 1$$

$$|\beta| = y_1$$

$$C\,[\text{dB}] = -20 \cdot \log_{10}(y_1)$$

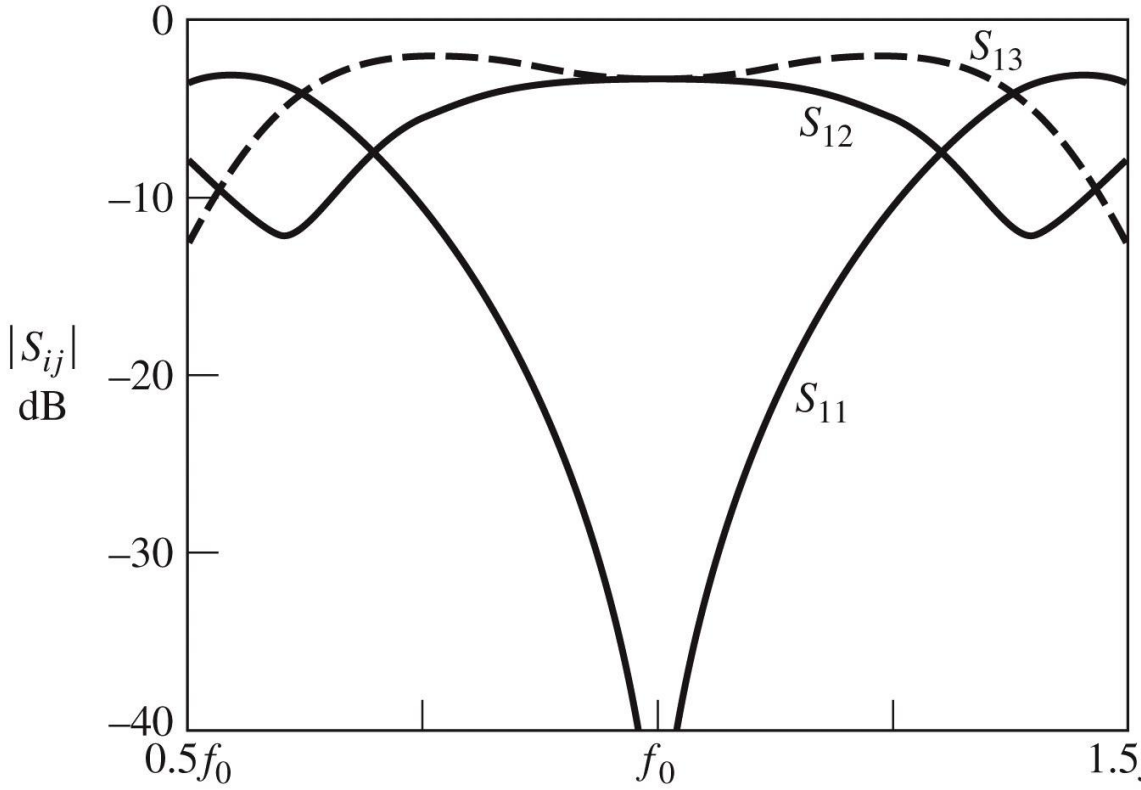


Figure 7.46
© John Wiley & Sons, Inc. All rights reserved.

Ring coupler

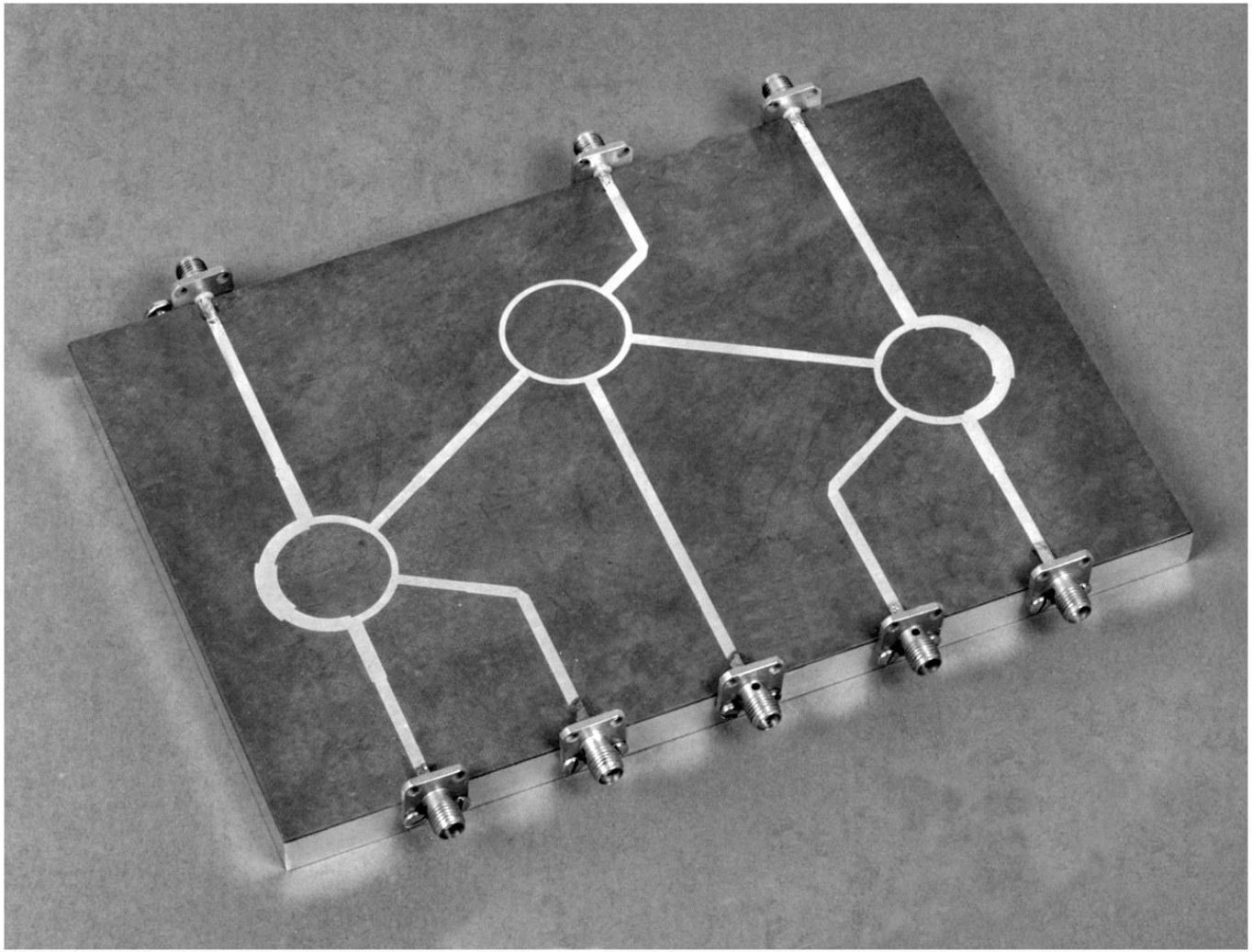
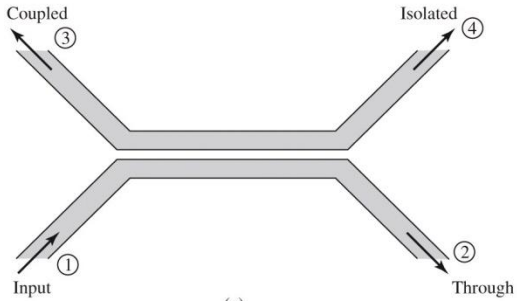


Figure 7.43
Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

Coupled Line Coupler



$$Z_{ce}Z_{co} = Z_0^2$$

$$|\beta| = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}$$

$$C\,[\text{dB}] = -20 \cdot \log_{10}\left(\frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}\right)$$

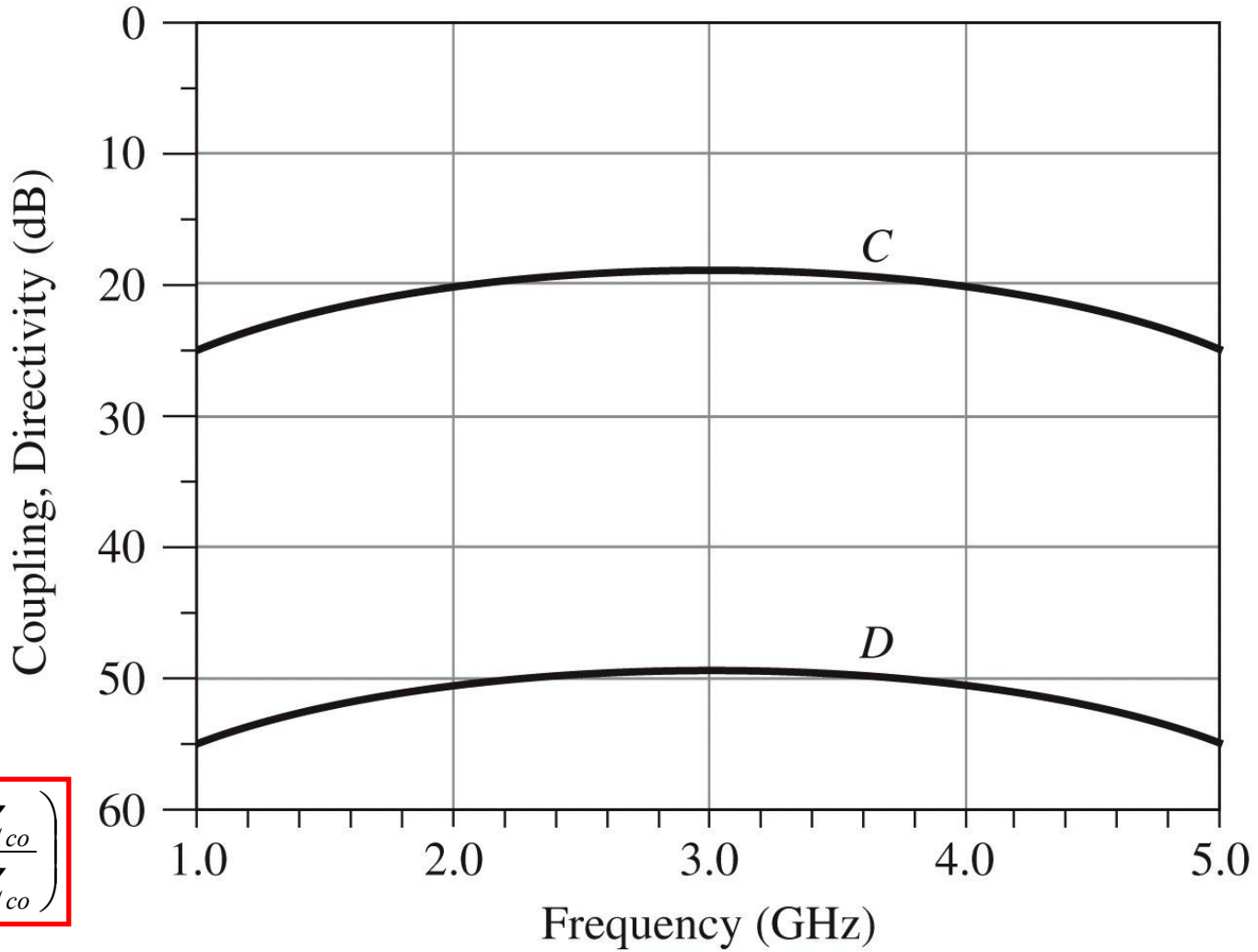
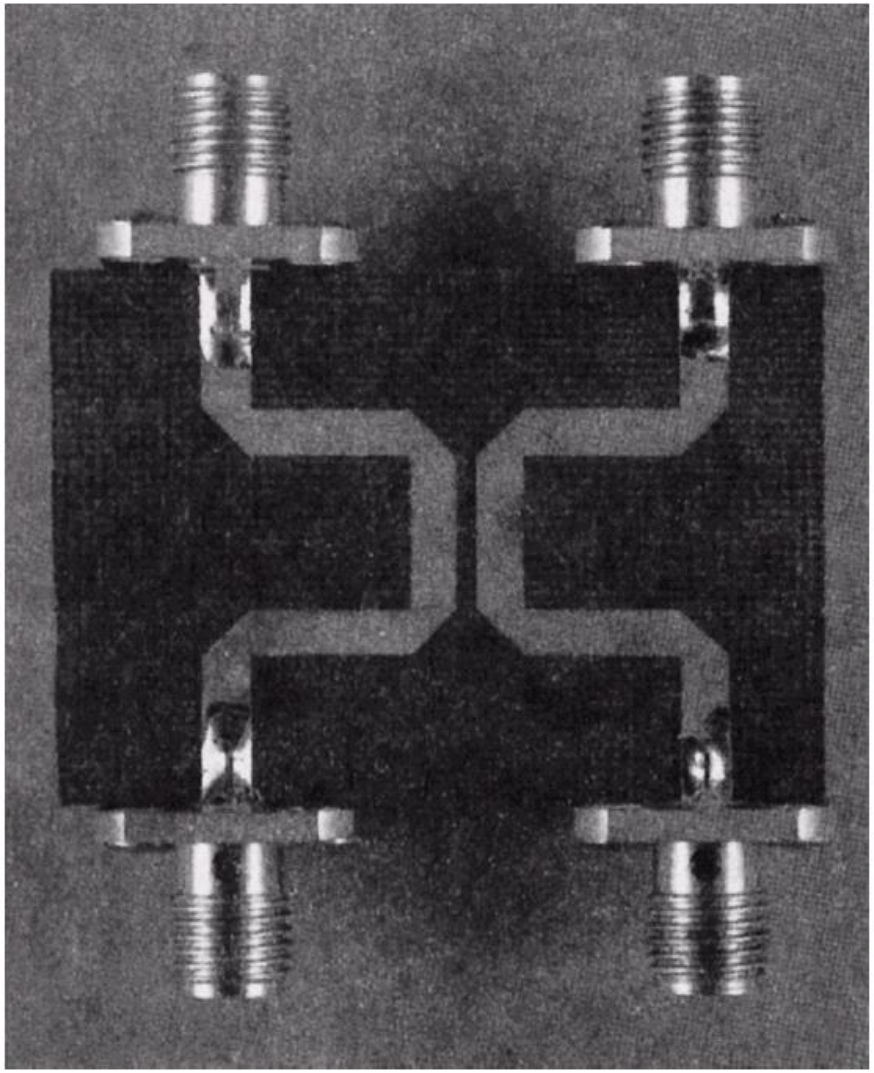


Figure 7.34

© John Wiley & Sons, Inc. All rights reserved.

Coupled line coupler



Contact

- Microwave and Optoelectronics Laboratory
- https://rf-opto.etti.tuiasi.ro
- rdamian@etti.tuiasi.ro